The paper starting on the next page proposes

Two Changes in the International System of Units

My second proposal has in its negative part (delete dimension one) been proposed before, but the first proposal (exchange amount of substance for number of elementary entities) is entirely original, even though it connects to some earlier made proposals for changes in the SI. However, the argumentation for both proposals brings some completely new arguments and points of view to the fore. Also, I make two claims that I have not seen discussed anywhere: (a) the mole is not a unit on a par with the other base units in the SI, and should be called a 'parameter unit'; (b) metrological multiplications of dimensions and of metrological units importantly differ from arithmetical multiplications. Nonetheless, *Metrologia*, the journal devoted to discussions of SI topics, has decided not to publish my paper (mail, April 3, 2009). The referees' reports, as well as my comments to them, can be found below – after my submitted paper. As can be seen from my comments, I find the reports very much wanting.

Since *Metrologia* is the only journal for discussion of SI proposals (philosophy of science journals have never papers concerned with the SI system), I am now (since April 26, 2009) using internet and my home site (Section 3) for trying to get my two combined proposals better scrutinized. That is, I would like anyone who has comments, be they negative or positive, to mail them to me (ingvar.johansson@philos.umu.se). My hope is that such comments will *either* convince me that my proposals really have better been dropped *or* encourage me to make, in some way or other, a second attempt to make the proposals visible to researchers interested in the overall viability and usefulness of the SI system.

Yours sincerely,
Ingvar J

Exchange *Amount of Substance* for *Number of Elementary Entities*, and Delete *Dimension One*

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Abstract

It is claimed that the SI ought to exchange the base quantity amount of substance for another quantity of the same dimension: *number of elementary entities*. Also, it is claimed that the notion of 'dimension one' is superfluous and can be taken away completely. Three things are central to the argumentation: (i) keeping in mind the difference between pure mathematical numbers and physical quantities; (ii) noting the difference between *count noun kinds* and properties such as length, mass, and duration; (iii) realizing that the mole is not an ordinary base unit, but a *parameter unit*.

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1. Two Proposals

In this paper two proposals for the SI are made and argued for. Both are related to, but different from, earlier proposals for changes that have been made in *Metrologia*. Most importantly, the argumentation is of a new kind, and one that makes the two proposals connected. The crucial notions to be introduced are ‘parameter unit’ and ‘count noun kind’.

The first proposal (A) is a kind of transformation of R. Dybkaer’s proposal that “‘Number of entities’ is dimensionally independent of the current base quantities and should take its rightful place among them” [1, p. 69]; and the proposal tries also in its own way to take account of what is said in this quotation from P. Becker et al:

> It is proposed that the Avogadro constant be converted to a number, the ‘Avogadro number’, and that the mole be linked to this number. The unit of the amount-of-substance would be this particular number of specified, identical entities. This would not only bring greater clarity and simplicity to the SI, but would also lead to a better understanding of the mole by the physics and chemistry communities, as well as by the general public. [2, p. 11]

My first proposal is:

(A) exchange the base quantity amount of substance for the quantity *number of elementary entities* (symbol *e-e*); and exchange the base unit mole (of kind E_n) with the base unit ^{e}*entity*$_n$ (symbol E_n).

The subscript *n* is meant to be a parameter for all the different kinds of elementary entities that the SI allows. Note that the proposal talks of ‘*elementary* entities’, not just ‘entities’. If accepted, the proposal has the consequence that the mole suffers the same fate in relation to $^{e}\text{entity}_n$ as the units minute, hour, and day have done in relation to second. That is, the mole becomes a non-SI unit accepted for use with the SI.

Superficially, it may look as if number of elementary entities cannot be a quantity of the same dimension as amount of substance, which the proposal presupposes, but a close look at the SI will remove this impression. Therefore, there is no need to propose a new symbol for the dimension number of elementary entities; the old N will do fine.

The proposed new base unit ^{e}entity$_n$ (symbol E_n) is not, it should be noted, a base unit on a par with m, kg, s, A, K, and cd; it is a parameter for base units. One contention of mine is that already the mole implicitly functions as such a *parameter unit*.

The second proposal (B) has its roots in reflections around the unit one and the dimension one such as those put forward by W. H. Emerson [3, 4, 5] and J. Valdés [6]. It says:

(B) delete dimension one completely, and with respect to derived dimensions of the form dimension$_D$ · dimension$_D^{-1}$ replace the unit one with the parameter unit: unit$_D$ · unit$_D^{-1}$ (symbol '$u_D \cdot u_D^{-1}$').

The subscript D is meant to be a parameter for all dimensions accepted by the SI. I hope to show that as soon as the notions of 'parameter unit' and 'count noun kind' are understood, the proposal B comes naturally.

2. Background to the Proposals

Two observations indicate that the SI is in need of improvement.

First Observation: the introduction of the base unit mole differs in structure from all the other six; it contains two paragraphs, the others only one (which corresponds to the first paragraph below). The SI Brochure says:

> 1. The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon 12; its symbol is "mol."
>
> 2. When the mole is used, the elementary entities must be specified and may be atoms, molecules, ions, electrons, other particles, or specified groups of such particles. [7, p. 115]

This means that, strictly speaking, the base quantity in question is not just amount of substance, but amount of substance *of specified elementary entity,* or more briefly: amount of substance of elementary entity E_n. Furthermore, since the number of atoms in 0.012 kilogram of carbon 12 (^{12}C) is known and equal to the Avogadro number (which I will symbolize A_N), the first paragraph above is logically equivalent to the one below:

> 1*. The mole is the amount of substance of a system which contains A_N elementary entities; its symbol is "mol."

And many scientists present the mole in this way. Here is a quotation from one well established introduction to physical chemistry: "A ***mole*** of some substance is defined as an amount of that substance which contains Avogadro's number of elementary units" [8, p. 9].

Note that I speak of the Avogadro *number* A_N (= 6.022 14 × 10^{23}) and not the Avogadro *constant* N_A (= 6.022 14 × 10^{23} mol^{-1}), which is a quantity. In my proposal, it is important to keep mathematical numbers and physical quantities (= mathematical number plus unit of a physical dimension) distinct. The relation between the Avogadro quantity (constant) and the Avogadro number is: $N_A = A_N$ mol^{-1}.

The importance of this distinction is easily shown. In statement (1*) the expression 'contains A_N elementary entities' cannot be exchanged for 'contains A_N mol^{-1} (N_A) elementary entities'; this would make the definition circular.

If the expression '1 mole of the amount of substance ^{12}C' is abbreviated into '1 mole amount of ^{12}C', and 'A_N number of elementary entities of kind atom ^{12}C' is abbreviated into 'A_N elementary entities ^{12}C', the SI implies the following equalities:

- 1 mole amount of ^{12}C = A_N elementary entities ^{12}C (which fits the general form:)
- 1 mole amount of E_n = A_N elementary entities $^{e}entity_n$ (which can be abbreviated into:)
- 1 mol E_n = A_N E_n

When the elementary entities spoken of in the definition of the mole become specified, which is required by the second paragraph of the SI definition, even *the mole itself becomes specified*. In the exemplification, the term 'mole' is turned into the expression 'mole of ^{12}C'. Quite clearly, even though only implicitly, the SI treats the mole as a parameter unit that has to be given a value before it can start to function as a real metrological unit. That is, 'mole' has ever since its introduction in the SI (1971) in practice been used as if it means 'mole of E_n'.

Proposal A implies that the statement '1 mole amount of E_n = A_N elementary entities $^{e}entity_n$' should in the future be regarded as being on a par with statements such as '1 minute duration = 60 seconds duration'. That is, the (parameter) unit $^{e}entity_n$ (E_n), is regarded as the accepted (parameter) base unit, and the mole is in general metrological contexts regarded as a somewhat obsolete unit for the same dimension. To repeat: *number of elementary entities* and *amount of substance* are different quantities of the same dimension, N.

I have taken the term ‘elementary entity’ from the SI, but, then, what kind of quantity is an elementary entity? If it is a derived quantity, how is it derived? And if it is not, why does the SI not list it as a base quantity? With these questions in mind, let me move to the second observation.

Second Observation: the SI lists only seven base quantities and base units [7, pp. 105, 111-116], but in passing there is also an eighth base quantity and base unit acknowledged; see the two quotations below (italics added):

> There are also some quantities that *cannot be described in terms of the seven base quantities of the SI at all*, but have the nature of a count. Examples are number of molecules […]. Such *counting quantities* are also usually regarded as dimensionless quantities, or quantities of dimension one, with the unit one, 1. [7, pp. 105-6]

> All of these *counting quantities* are also described as being dimensionless, or of dimension one, and are taken to have the SI unit one, although the unit of counting quantities cannot be described as a derived unit expressed in terms of the base units of the SI. For such quantities, *the unit one may instead be regarded as a further base unit*. [7, p. 120]

Hereby, the SI says that it accepts an eighth base quantity, which is either of the (so to speak) dimension *dimensionless* or of the dimension *dimension one*; base unit *one*, symbol 1. And more or less the same thing is said in VIM (italics changed):

> For *number of entities*, the number one, symbol 1, can be regarded as a *base unit* in any system of units. [9, p. 7][1]

In these quotations, the expressions ‘counting quantities’ and ‘number of entities’ (which I take to be different names for the same thing) refer to a presumed *base* quantity. Since ‘number of entities’ subsumes ‘number of *elementary* entities’, the quotations seem to answer my earlier question. *Elementary entity* is in the SI regarded as a base quantity, but it is regarded as being of such a character (having “*the nature of a count*”) that it needs not to be listed in the SI. Nonetheless it is ascribed a metrological unit, one, that is regarded as being the unit for some quantities that in the SI are derived quantities. The expressions ‘counting quantities’ and ‘number of entities’ seem to refer to something that is at one and the same time both outside and inside of the traditional seven-base-units system. My proposals take this oddity away.

The content of the last three quotations is in conformity with some papers that have been published in *Metrologia*; in particular Mills [10] and Dybkaer [1]. Dybkaer puts forward a proposal that must not be conflated with mine, even though mine is partly inspired by his. He argues that the SI system should be *enlarged* with an eighth base quantity called ‘number of entities’, whereas I want the quantity called ‘number of *elementary* entities’ to *replace* amount of substance.

Mills and Dybkaer are in favor of the SI dimension *dimension one* and its unit *one*, but it should be noted that this dimension and unit are not in general regarded as unproblematic. In particular, they have been criticized by Emerson [3, 4, 5] and Valdés [6], with whose criticism

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[1] I disregard the peculiarity that the quotation seems to identify a pure mathematical number with a metrological unit. It makes more sense to write: For number of entities, the *metrological unit one*, symbol 1, can be regarded as a base unit in any system of units.

I now mainly agree.[2] Nonetheless, I do *not* endorse their positive view, which is this: when a dimension or metrological unit is divided by itself we obtain only a pure mathematical number. As I will show in the next section, there is a third option beside what I will call 'the unit one view' of the SI and 'the pure number view' of the critics. Let me now explain all this and unpack my proposal in more detail.

3. Numbers, Quantities, Count Noun Kinds, and Metrological Multiplications

I will now start from scratch with the distinction between pure mathematical numbers and quantities. To be somewhat clear about this distinction is a necessary presupposition for understanding the proposals.

If one is asked 'please, point at the pure mathematical number 1', one does not at all know what to point at. In this sense (at least), we do never in the spatiotemporal world meet any pure numbers. Nonetheless, we meet something related. Instead of pure numbers we can meet unities of a certain *kind*, i.e., quantities. Terms such as '1 pebble', '2 chairs', '3 flowers', '4 horses', '5 birds', '6 atoms ^{12}C', '7 water molecules', etc. can be used to refer to entities in the world. And there is no problem in pointing at, for instance, 1 pebble. Bringing in SI terminology one can say, either that kinds of things such as pebbles, chairs, and molecules *are their own metrological units*, or that in relation to such kinds *there is no distinction to be made between a dimension and its metrological unit*. Linguists call terms of the kind now used 'count nouns', and I will call the entities referred to 'count noun kinds'.

[2] I have not always done so. Not long ago, I put forward a proposal [11] that is similar to the proposal A of this paper, but which retained the metrological unit one; and this makes quite a difference.

Only entities of the same count noun kind can in a meaningful way be added. Additions such as ‘4 horses + 5 birds’ and ‘6 atoms + 7 molecules’ have no sum that makes sense. However, on a more abstract level a corresponding addition is possible. Since both horses and birds are animals, and ‘animal’ is a count noun, they can *as animals* be added: 4 animals + 5 animals = 9 animals. Similarly, since both atoms and molecules are elementary entities, and ‘elementary entity’ is a count noun, atoms and molecules can *as elementary entities* be added: 6 elementary entities + 7 elementary entities = 13 elementary entities.

Now, climbing the ladder of abstraction to the very top, we encounter the most abstract count noun kind possible: *entity*. Classified only *as entities*, everything whatsoever can be added. For instance, horses, molecules, nations, dreams, and instances of pain: 4 entities (horses) + 5 entities (molecules) + 21 entities (nations) + 2 entities (dreams) + 13 entities (instances of pain) = 45 entities.

Such abstract entity additions, however, are of no more practical use than additions of pure mathematical numbers, since on this topmost abstract level no kinds of things are differentiated from other kinds of things.[3] Therefore, there is no need to add any base quantity *number of entities* to the SI. Such a quantity (having “*the nature of a count*”) will have no function that is not already taken care of by the pure mathematical numbers.

Terms such as ‘water’, ‘snow’, and ‘furniture’, do not function the way count nouns do; they cannot immediately in a meaningful way be connected to a numeral. Expressions such as ‘2 water’, ‘3 snow’, or ‘4 furniture’ make no sense, and linguists have baptized these terms ‘mass nouns’.[4] What mass nouns refer to cannot without further ado be counted. In order to

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[3] It might be argued that, in fact, the so-called *pure mathematical numbers* 1, 2, 3, etc. are nothing but the *most abstract quantities* possible, namely 1 entity, 2 entities, 3 entities, etc., but I leave this subtle question to the philosophy of mathematics.

[4] This term ‘mass’ has of course nothing with the SI base quantity mass to do.

estimate, for instance, amount of water, one has to introduce a unit such as ‘bottle’, ‘glass’, or ‘molecule’. The expressions ‘bottle of water’, ‘glass of water’, and ‘molecule of water’ function the way count nouns do; by means of bottles, glasses, and molecules different amounts of water can be estimated. Out of ‘furniture’ the term ‘furniture item’ can be created, and as soon as it is decided what counts as a furniture item, then even amount of furniture can be estimated.

Terms for physical-chemical substances such as ‘^{12}C’ and ‘H_2O’ are out of context ambiguous between being mass nouns and count nouns. But always when they are shorthand for ‘^{12}C atom’ and ‘H_2O molecule’, respectively, they are count nouns, and refer to count noun kinds that *are their own metrological units*. This means, among other things, that their amount can be estimated independently of any introduction of the mole.

All the terms for base *dimensions* in the contemporary SI function the way mass nouns do. Expressions such as ‘1 length’, ‘1 mass’, ‘1 temporal duration’, and ‘1 amount of substance’ are just as meaningless as ‘1 water’ and ‘1 snow’. But all the terms for base *units*, e.g., ‘meter’, ‘kilogram’, ‘second’, and ‘mole’ function the way count nouns do, and refer to count noun kinds. Expressions such as ‘1 meter’, ‘2 kilogram’, ‘3 seconds’, and ‘4 mole’ make perfect sense. This is the reason why *the base units of the SI are their own metrological units*. In order to be used in practice, no base unit in the SI needs to be connected to a further metrological unit such as the unit one.

I hope these remarks are enough to sustain the following view of mine: to claim that the dimension number of entities needs the metrological unit one, is on a par with, and as unreasonable as, claiming that the base units of the SI have to be connected to the metrological unit one.

The quantity *number of elementary entities*, which I want to introduce, is a specific case of the quantity *number of entities*. This means that what I have just said about the latter quantity holds for the former, too. *Elementary entity* is a count noun kind, and therefore not in need of any special metrological unit before it can be put to estimating-quantity use. What is needed is only a delineation of what should count as an elementary entity. In this respect, however, I need to say no more than what is already said in the SI: elementary entities "may be atoms, molecules, ions, electrons, other particles, or specified groups of such particles".

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Above, I have shown that the quantity *number of elementary entities* is its own metrological unit and not in need of any *dimension one* or special unit *one*. But this fact does not in itself imply that the notion of 'dimension one' ought to be taken away from the SI, as proposal B suggests. It might still be needed in relation to *derived* quantities whose dimension is $\text{dimension}_D \cdot \text{dimension}_D^{-1}$ and whose unit is $\text{unit}_D \cdot \text{unit}_D^{-1}$, i.e., needed in relation to quantities such as radian ($m \cdot m^{-1}$) and mass fraction ($kg \cdot kg^{-1}$). I now turn to an analysis of this kind of derived quantities.

Before looking at quantities of the form $x\ \text{unit}_D \cdot y\ \text{unit}_D^{-1}$ (examples: $3m \cdot 5m^{-1}$ and $4kg \cdot 7kg^{-1}$) we should be clear about what is multiplied in both $x\ \text{unit}_D \cdot y\ \text{unit}_D$' (example: $3m \cdot 5m$) and $x\ \text{unit}_{D1} \cdot y\ \text{unit}_{D2}$ (example: $3s \cdot 5A$). Explicitly mentioned in the multipliers and the multiplicands are only one numeral and one metrological unit, but implicitly there is a third entity present, namely the *dimension* that the unit is the metrological unit *of*. When the dimensions are made explicit, we obtain expressions such as '3m *length* · 5m *length*' and '3s *duration* · 5A *electric current*', respectively. Using the SI symbols of the dimensions, we can write the last two expressions as '3m L · 5m L' and '3s T · 5A I'. If there were no dimensions implicitly present in quantity expressions, then statements such as '1 meter = 1.0936 yards', which relate to each other different units of the same dimension, would make no sense. But they do.

In the equality 3m L · 5m L = 15m^2 L^2, as well as in 3s T · 5A I = 15C T I, and in similar ones, we have to distinguish between the arithmetic multiplication of the pure mathematical numbers (3 · 5 = 15) and the *metrological multiplications* of both the dimensions (L · L = L^2; T · I = T I = Q) and the metrological units (m · m = m^2; s · A = s A = C). In what follows, it does not matter whether I exemplify metrological multiplication with multiplication of dimensions or metrological units. Since they work in tandem, the reasoning will be the same. What I will stress is this:

- metrological multiplication importantly differs from arithmetic multiplication.

Arithmetic multiplications of integers (which I think is enough to discuss here) have always a very clear-cut connection to arithmetic addition; a multiplication can be regarded as repeated addition. Metrological multiplications, however, have no relations to any corresponding metrological additions. Furthermore, they cannot have, since there simply is no such thing as meaningful additions of physical dimensions or metrological units. For instance, whereas the multiplication 3 · 5 is equivalent to the repeated addition 3 + 3 + 3 + 3 + 3 (or 5 + 5 + 5), neither for m · m (L · L) nor for s · A (T · I) is there any corresponding metrological addition. In additions such as 3m + 3m + 3m + 3m + 3m the metrological units are *not* added. In the exemplification, five quantities *of the same dimension and metrological unit* are added, but the dimensions and units themselves are not added.

This difference between arithmetic and metrological multiplication means that it cannot be taken for granted that what is true of arithmetic multiplications ($x \cdot x^{-1} = 1$) must also be true of metrological multiplications. That is, that in some sense $D \cdot D^{-1} = 1$ and $\text{unit}_D \cdot \text{unit}_D^{-1} = 1$. However, this is exactly what seems to be taken for granted in the SI (italics added):

> Certain quantities are defined as the ratios of two quantities of the same kind, and are *thus* dimensionless, or have a dimension that may be expressed by the number one. The coherent SI unit of all such dimensionless quantities, or quantities of dimension one, is the number one, since the unit *must be* the *ratio* of two identical SI units. The values of all such quantities are simply expressed as numbers, and *the unit one is not explicitly shown*. [7, 120]

(Except for the parenthesis in the second next paragraph, I will disregard the term 'dimensionless', which in the quotation is treated as being synonymous to 'dimension one'.) According to the quotation, it holds true that $D \cdot D^{-1}$ = dimension one, and that $\text{unit}_D \cdot \text{unit}_D^{-1}$ = unit one. I will call this view 'the unit one view of same-dimension division'. I have already shown that the last two equalities by no means can be regarded as self-evident truths, and this fact strongly reinforces a peculiarity that has been remarked on before: it seems odd that quantities as different as mass fraction and radian should be regarded as quantities of exactly the same dimension, not to speak of area and Reynolds number [4, p. L27, n1], and many other similar examples. In the future, I think it befalls on the defenders of the unit one view to show that $D \cdot D^{-1}$ = dimension one and that $\text{unit}_D \cdot \text{unit}_D^{-1}$ = unit-one. The burden of proof is on them, not as hitherto, on the critics.

Normally, the critics claim that $\text{unit}_D \cdot \text{unit}_D^{-1}$ is not equal to unit one, but to the pure mathematical number 1. That is: $\text{unit}_D \cdot \text{unit}_D^{-1}$ = (number) 1. I will call this view 'the pure number view of same-dimension division'. Now, if there is no metrological unit, there is no SI dimension either. Therefore, consequently, away with the dimension dimension one, too. (On this view, it can be noted, it makes very good sense to say that the result of the metrological operation $\text{dimension}_D \cdot \text{dimension}_D^{-1}$ is something dimensionless.) The oddity of this view is that it takes it for granted that a metrological multiplication can give a nonmetrological purely arithmetic result. Out of both $D \cdot D^{-1}$ and $unit_D \cdot unit_D^{-1}$ come, the critics claim, only the pure number 1. Since I have seen no discussion of how metrological multiplication can result in pure numbers, it seems to me as if the pure number view gets its reasonableness not from a good analysis of metrological multiplication, but from the oddity of the unit one view.

Now, if these two views were the only possible ones, the choice would be hard, but I happen to think that there is a third and better view available. I will call it 'the parameter unit view of same-dimension division'; and I will soon present it. Let me first only display the three views (of same-dimension division) line by line:

i. the unit one view: $unit_D \cdot unit_D^{-1}$ = (metrological unit) 1

ii. the pure number view: $unit_D \cdot unit_D^{-1}$ = (mathematical number) 1

iii. the parameter unit view: (unit) one $\neq$ $unit_D \cdot unit_D^{-1}$ $\neq$ (number) 1

According to the third option, the metrological unit of mass fraction can be nothing else than the unit: mass (of part) · mass (of whole)$^{-1}$. Similarly, the unit of radian can be nothing else than length (of circle arc) · length (of radius)$^{-1}$. This view is also intended to mean that a metrological multiplication such as $unit_D^3 \cdot unit_D^{-2}$ is not equal to $unit_D$, but can be nothing else than the un-reducible $unit_D^3 \cdot unit_D^{-2}$. That is: $unit_D^3 \cdot unit_D^{-2} \neq unit_D$. To exemplify, the metrological unit for rainfall is volume per area ($m^3 \cdot m^{-2}$) and nothing else, not length (m).

First now to be noted is the fact that all metrological units such as $m \cdot m^{-1}$ and $m^3 \cdot m^{-2}$ are count noun entities; just like the base units of the SI and the count noun kinds discussed in relation to the mole are. Therefore, they can at once function as metrological units without any help from some further metrological unit. This, by the way, is also true of units of the form $unit_{D1} \cdot unit_{D2}$.

Second, it may seem as if regarding units of the form $\text{unit}_D \cdot \text{unit}_D^{-1}$ as un-reducible units would imply an endless proliferation of units that correspond to the old unit one. In a sense, of course, this is the case, but it is an innocent sense. In specific scientific contexts, there are always only a quite restricted number of units of the form $\text{unit}_D \cdot \text{unit}_D^{-1}$, and they can in the context at hand be handled as they are. And in general metrological contexts, where this might not be possible, proposal B introduces the parameter unit $\text{unit}_D \cdot \text{unit}_D^{-1}$.

Third, metrological multiplications are now and then used for checking physical-chemical calculations: a necessary requirement for a quantity equality to be true is that both sides of the equality have the same physical dimension. Nothing in proposal B takes this practical use of metrological multiplication away. When making such a check, one may well for simplicity's sake substitute 1 for $\text{unit}_D \cdot \text{unit}_D^{-1}$, and unit_D for $\text{unit}_D^{3} \cdot \text{unit}_D^{-2}$; be it then only remembered that such substitutions are only simplifying and useful mnemonic devices in the check. In this sense, I have no objections to equalities such as: $m^2\ m^{-2}\ kg\ s^{-3} = kg\ s^{-3}$ [7, p. 119].

In sum, the unit one ought to be exchanged for the parameter unit $\text{unit}_D \cdot \text{unit}_D^{-1}$, and this unit is not in need of any special dimension of its own; for each parameter value the correspondingly derived dimension will be the dimension needed.

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We can now clearly see that both proposals rely on the notions of 'count noun kind' and 'parameter unit'. In Proposal A, the elementary entities mentioned have to be regarded as count noun kinds, and the base unit E_n as a parameter unit. In Proposal B, it is presupposed that all derived units function as count noun kinds, and that the unit $\text{unit}_D \cdot \text{unit}_D^{-1}$ is a parameter unit.

4. Conclusion

I would like Table 1 in the SI Brochure (section 2.1.2) [7, 116] to be exchanged for the table below. There is no need to find a new symbol for the dimension of the new quantity *number of elementary entities*, since its dimension is the same as that of amount of substance (with dimension symbol 'N'):

Base quantity		SI base unit	
Name	Symbol	Name	Symbol
length	*l*, *x*, *r*, etc	metre	m
mass	*m*	kilogram	kg
time, duration	*t*	second	s
electric current	*I*, *i*	ampere	A
thermodynamic temperature	*T*	kelvin	K
number of elementary entities	***e-e***	^{e}entity$_{n}$	E_n
luminous intensity	I_v	candela	cd

References

[1] Dybkaer R 2004 Units for quantities of dimension one *Metrologia* **41** 69-73

[2] Becker P, De Bièvre P, Fuji K, Glaeser M, Inglis B, Luebbig H, and Mana G 2007 Considerations on future redefinitions of the kilogram, the mole and of other units *Metrologia* **44** 1-14

[3] Emerson W H 2002 A reply to “Definitions of the units radian, neper, bel and decibel” by I M Mills et al. *Metrologia* **39** 105-9

[4] Emerson W H 2004 One as a ‘unit’ in expressing the magnitudes of quantities *Metrologia* **41** L26-8

[5] Emerson W H 2004 On the algebra of quantities and their units *Metrologia* **41** L33-7

[6] Valdés J 2002 The unit one, the neper, the bel and the future of the SI *Metrologia* **39** 543-9

[7] *The International System of Units (SI)* 8th edition 2006 (Paris: Bureau International des Poids et Mesures) <http://www.bipm.org/en/si/si_brochure/>

[8] Levine I N 1983 *Physical Chemistry* (New York: McGraw-Hill)

[9] *International Vocabulary of Metrology – Basic and General Concepts and Associated Terms, VIM* 3rd edition JCGM 200: 2008 (Paris: Bureau International des Poids et Mesures) <http://www.bipm.org/en/publications/guides/vim.html>

[10] Mills I M 1994/95 Unity as Unit *Metrologia* **31** 537-41

[11] Johansson I 2008 Functions and Shapes in the Light of the International System of Units *Metaphysica* **9** 93-117

First Referees Report

The paper deals in a clear and generally convincing way with an important and controversial topic.
My comment:
Nice to hear!

Some minor points follows.
1. While the paper is well structured, the first pages of Section 3 contain nothing really new, and their contents could be synthesized / shortened, by also explicitly referring to one or more texts / papers on linguistics (e.g., Google search "grammar of measurement").
My comment:
Parts of Section 3 are meant only as an introduction for some readers and as memory refreshment for others; the two linguistic terms are in more detail explained in, for instance, Wikipedia. With respect to the referee's view that the pages in question "contain nothing really new", I would like to point out that these pages the following critical statement about the SI can be found: "to claim that the dimension number of entities needs the metrological unit one, is on a par with, and as unreasonable as, claiming that the base units of the SI have to be connected to the metrological unit one (p. 10)." I have never seen this or a corresponding statement made anywhere else.

2. All over the text the concept that the concerned entities must be "elementary" is emphasized (e.g., "The quantity number of elementary entities, which I want to introduce, is a specific case of the quantity number of entities."). On the other hand, this claim is not justified, and the assumed specific concept of "elementariness", if any, is not discussed.
My comment:
I thought that the concept of "elementary entity" was in no need of discussion, since it is the same as the one that is used in the SI; this becomes clear on p. 4 (point 2). Furthermore, somewhat later I explicitly say: "I have taken the term 'elementary entity' from the SI (p. 6, first line)."

3. The concept of "metrological multiplication", as in "Metrological multiplications, however, have no relations to any corresponding metrological additions" (p.12) is dealt with a little bit trivially. For example, I suppose that the same conclusions would be more generally and clearly justified in terms of scale transformation invariance, as in the Stevens' theory of scale types, and by referring to Bridgman's dimensional analysis.
My comment:
Yes, it is trivial, but my implicit point is of course that this triviality is a blind spot in the SI, which when seen implies the kind of observations that I make. I know much of the literature on scales, but one shouldn't complicate things that are not in need of complication.

4. The symbol E_n is used (p.2) before being explained (p.3). Furthermore, while at p.2 a distinction is made relating to its font style (regular for “kind”, italic for base unit), its occurrences throughout the text are in regular style, even when italic seems to be required. The symbol e-e is introduced (p.2) but then used only in the table in Section 4 (p.16).
My comment:
In my opinion and experience, sometimes the reading becomes easier if a new symbol is used once before it is explained. The second complaint I don’t understand. In the proposal (A), I am using regular font for traditional notation and italic font for the new terms in my proposal (both for ‘kind’ and ‘unit’).

5. Here and there in the text the expression “mathematical number” is adopted. While this seems to be done for emphasizing the contrast with “physical quantity”, the very concept of mathematical number is a little bit peculiar: do non-mathematical number exist at all?
My comment:
In my opinion, all numbers are mathematical numbers, but in order that no one should interpret me otherwise, I used the whole expression ‘mathematical number’. This is not linguistically odd; often, everyday language contains redundancy, and redundancy is better than risk of misinterpretation.

6. P.5: “... the (parameter) unit $^{e}\text{entity}_n$ (E_n), is regarded as ...”: remove the comma.
My comment:
I agree; the comma shouldn’t be there.

Second Referees Report

Abstract: Change “It is claimed that...” for “I claim that ...”, throughout.
(The author should make it clear who is making the claim; as it reads at present it appears as though he is reporting a claim made by someone else, on which he is going to comment.)
My comment:
I mistakenly thought it was obvious to anyone that in an abstract “It is claimed that” is short for “It is in the paper claimed that”.

Much of this paper is quite hard to follow, owing to the introduction of new names and symbols for quantities that he introduces (e.g. *parameter unit*, *count noun kind*, etc.) and units (e.g. $^{e}\text{entity}_n$ - if the complexity of this symbol is necessary it is not explained).
My comment:
In order for my proposals to contain names and symbols in the way that is standard in the SI system, the new names and symbols are necessary. I mistakenly thought that every reader should find it obvious that $^{e}\text{entity}_n$ is short for “elementary entity of kind n”.

Also much of this paper seems to display a lack of understanding of the system of quantities and units that forms the present basis of the SI.

My comment:

I am sure that (after some hard work) I now understand the structure of the present SI system and its complications. However, what might give the referee the contrary impression is, that I am looking at the SI system from the perspective of the notions of 'count noun kind' and 'parameter unit', which he or she never discusses.

In the current system, which is the basis of the SI, we use the quantity 'number of entities', symbol *N*, which is dimensionless (or as some prefer to say is a quantity of dimension 1).

My comment:

It is not just that "some prefer to say is a quantity of dimension 1"; in the SI the expressions 'dimensionless' and 'dimension 1' are used as synonyms; see quotations on p. 6 above.

We also use the quantity 'amount of substance' for which I shall write 'amount' for brevity, symbol *n*, which is a base quantity which thus has its own dimension, and for which the corresponding base unit is the mole, symbol mol. We can then write an equation like

(1) $PV = nRT$

where the SI units for the quantities are

P: pressure, unit: Pa = N m^{-2}

V: volume, unit: m^3 or dm^3

n: amount, unit: mol

My comment:

There is according to my proposal no problem in continuing using the mole and mol. In my proposal the following equality always holds true: n mol E_n = e-e ^{e}entity$_n$ A_N E_n (or briefly: n mol = e-e ^{e}entity$_n$ A_N). Note that the number n and the subscript n in E_n have nothing at all in common. A_N is the Avogadro *number*; the *constant* $N_A = A_N$ mol^{-1} (p. 4).

R: gas constant, unit: J mol^{-1} K^{-1} = N m mol^{-1} K^{-1}

In my proposal this equality becomes: J (^{e}entity$_n$ A_N)$^{-1}$ K^{-1} = N m (^{e}entity$_n$ A_N)$^{-1}$ K^{-1}

T: temperature, unit : K

We can also write essentially the same equation in the form

(2) $PV = NkT$

where

N: number of entities, unit 1

In my proposal the expression unit-1 is in relation to the example above deleted (with respect to derived dimensions it is substituted by the parameter unit: unit$_{D1}$ · unit$_{D2}$), which means that N unit-1 becomes simply e-e.

k: Boltzmann constant, unit: J K^{-1}

We then have the relations (3) $n = N/N_A$

My comment:

The equality (3) above is short for: n mol = N unit-1 / N_A (which can be continued as follows) = N unit-1 / A_N mol^{-1} = N unit-1 · A_N mol.

Since in my proposals (a) 'n mol' corresponds to 'e-e ^{e}entity$_n$ A_N' and (b) 'N unit-1' is substituted by 'e-e', the equality (3) is in my proposal transformed into the following equivalent equality:

e-e ^{e}entity$_n$ A_N = e-e ^{e}entity$_n$ A_N.

This makes obvious the tautological character of $n = N/N_A$. This equality is no natural law; it merely reflects the terminological fact that n by stipulation is tied to mol and N by stipulation to unit-1.

and (4) $R = k N_A$

We can also write relations such as (5) $c = n/V$,
i.e. concentration (amount concentration) = amount per volume of solution,
with the units: (mol/dm^3) on the left and (mol)/(dm^3) on the right.

Similarly we can write (6) $C = N/V$
where C denotes entity concentration, with unit (1/dm^3), and N has unit (1).

Chemists wish to retain the ability to use all these equations, with the units indicated. They wish to talk of a solution of concentration $c =$ 0.1 mol/dm^3, and at other times they wish to talk of a concentration $C =$ 10^6 molecules/cm^3. The author of this paper wishes to abolish the quantity amount of substance, with its own dimension, and replace it with number of entities on all occasions.

My comment:

No, I do not want to "replace [the mole] on all occasions". I am explicitly saying: "the proposal has the consequence that the mole suffers the same fate in relation to ^{e}entity$_n$ as the units minute, hour, and day have done in relation to second. That is, *the mole becomes a non-SI unit accepted for use with the SI* (p. 2)." I am both well aware of and accept the fact that tradition and pragmatic considerations have to play a role in the SI system, too.

It would follow that we would not need both the gas constant and the Boltzmann constant – we should dispense with one of them (which?),

My comment:

My proposal does neither introduce nor abolish the use of the equality implicitly presented by the referee, namely: nRT = NkT. As far as I am concerned, chemists can very well continue to use both the constants R and k.

and similarly we would not need both the Faraday constant and the elementary charge (which appear in the equation (7) $F = N_A e$).

The thing to note is that current system, described briefly above, is the current language of chemistry, which has grown up over many years. Chemists are used to it; they find it convenient; and they do not wish to abandon it. The proposal to abolish the quantity amount of substance and use only number of entities has been made many times before (e.g. by Emerson, and Becker, referred to in this manuscript). There is a logical case for this proposal, but chemists do not want it (and nor do I want it!).

My comment:
The SI system is meant for the whole natural-scientific community, not only for chemists. And the mole suffers from kinds of misinterpretations that no other base unit suffers from. For instance, as late as February 2008 Wikipedia falsely said (but now corrected) that "A mole is much like 'a dozen' in that both are absolute numbers (having no units)", and the computer dictionary 'Wahtis.com' said (and still says, April 2009) that "The mole is the only fundamental SI unit that is dimensionless"; both facts reported in my paper [11]. In the paper "Amount of substance and the mole" (*Physics Education*, July 1977, 276-78), the chemist M L McGlashan found it necessary to write a paper in order to try to correct wrong interpretations of the mole. I quote:

"Although widely used by chemists, the physical quantity called amount of substance and its SI unit called the mole are not necessary in science. It would be perfectly feasible to deal always with molecular quantities and at least in physics that is often done. [---] Nevertheless, for historical reason it is customary in chemistry (and in physics too) to use the redundant physical quantity amount of substance and its SI unit the mole. So long as they are used they should be used correctly. The rest of this article will be about their correct use."

To sum up, there is a sort of philosophical logic in this paper, based on somewhat similar proposals that have been made before. They have never been adopted, and the proposals in this paper will never be adopted, because the users of the quantity 'amount of substance' and its unit 'mole', who are primarily chemists, like the present system.
My comment:
The proposals in question have so far not been adopted, but – note – they have not been rejected after extensive discussion. At least in *Metrologia*, none of the proposals mentioned in my paper is more than seven years old. Discussions to change the SI system need time, and I happen to add some completely new arguments to the hitherto embryonic *Metrologia* discussion.

I have to also comment that it is a struggle to read this paper, at least I find it so. It would be even more of a struggle for the average analytical chemist,
My comment:
Yes, both because they will meet new terminology, and because they will have to enter an abstract problem area they have not trod before. This fact, however, means only that they will have to spend more time with this paper than with the chemistry papers they normally read. On the other hand, if my proposals become accepted, there will be no more problems in teaching the modified SI system to students than it nowadays is to teach them the present one.

who are big users of the current system. For all these reasons I am not particularly in favour of publishing it, and I think it will not be much read if you do publish it. But it is I suppose presenting a logical alternative system, although it is not going anywhere in my opinion.
My comment:
Good to hear that my proposal is at least regarded as a coherent system.

Editors Report

The referees have mixed views on this article. However, on the basis of the two reports I am unable to accept this article for publication in Metrologia.
Referee 1 - although basically favourable - finds that the article is too long and rather confusing.
Referee 2, on the other hand, finds that the article will not advance the field in practice. I wonder if it might be better suited to a more philosophical review.
My comment:
As far as I can see, I am one of very few philosophers that have paid serious and intense attention to the foundational structure of the SI system, and I am fairly sure that my paper will receive no attention in an ordinary philosophy of science journal.