## 5. Beyond first-order logic

Many logicians would contend that there is no logic beyond first-order logic, in the sense that when one is forced to make all one's mathematical (extra-logical) assumptions explicit, these axioms can always be expressed in first-order logic, and that the informal notion of *provable* used in mathematics is made precise by the formal notion *provable in first-order logic*. Following a sugestion of Martin Davis, we refer to this view as *Hilbert's Thesis*.

The first part of Hilbert's Thesis, that all of classical mathematics is ultimately expressible in first-order logic, is supported by empirical evidence. It would indeed be revolutionary were someone able to introduce a new notion which was obviously part of logic. The second part of Hilbert's Thesis would seem to follow from the first part and Gödel's Completeness Theorem. Thus Hilbert's Thesis is, to some extent, accepted by many mathematical logicians.

Even those who accept Hilbert's Thesis in theory, however, are a far cry from accepting it in practice. It would be completely impractical and, in fact, counter-productive, to always make all one's extra-logical assumptions explicit.