The Logic of Extensional RDFS

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Summary

- Normative RDFS:
  - unexpected behaviour
    - incompatible with the expected semantics.
- The fixes in the spec by Pat Hayes are not enough.
- Extensional RDFS:
  - with the expected behaviour
    - compatible with the set-based and OWL semantics.
- Reasoning with Extensional RDFS:
  - same complexity and implementations as normative RDFS;
  - non-trivial proof.

The ϱdf fragment of RDFS

- ***ϱdf*** includes only the special RDFS vocabulary:
  - **rdf:type**
  - **rdfs:subclass**
  - **rdfs:subproperty**
  - **rdfs:domain**
  - **rdfs:range**

- ***ϱdf*** is self-contained:
  - it does not depend on the RDFS vocabulary beyond this subset.

Intended meaning of RDFS

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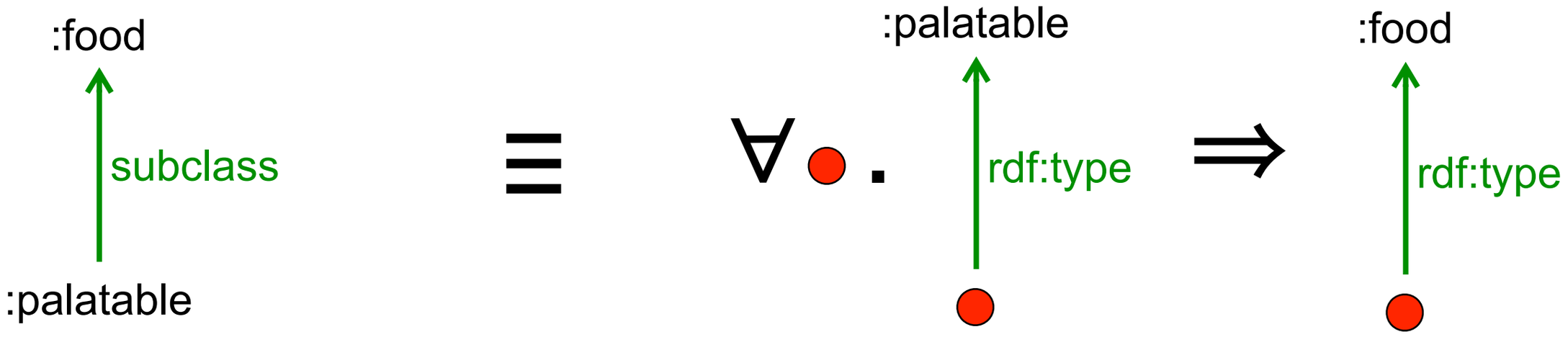

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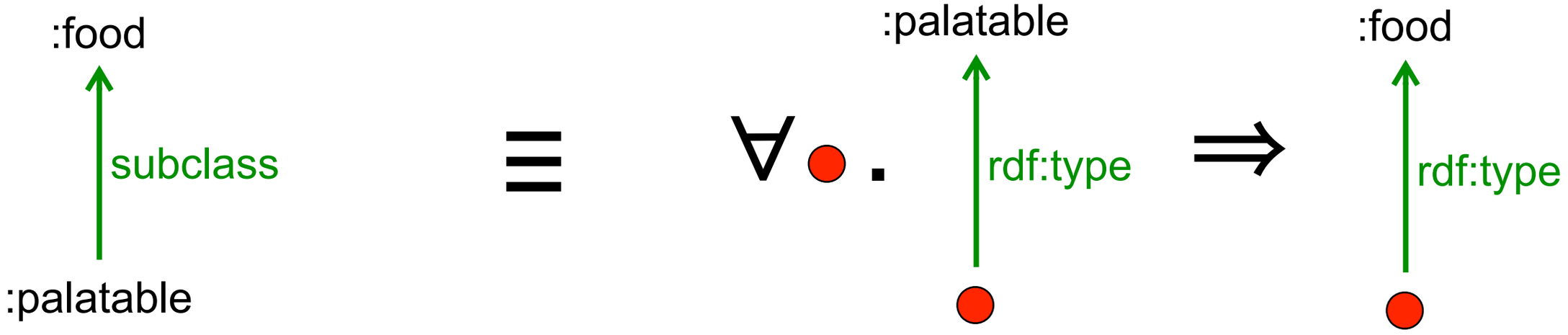

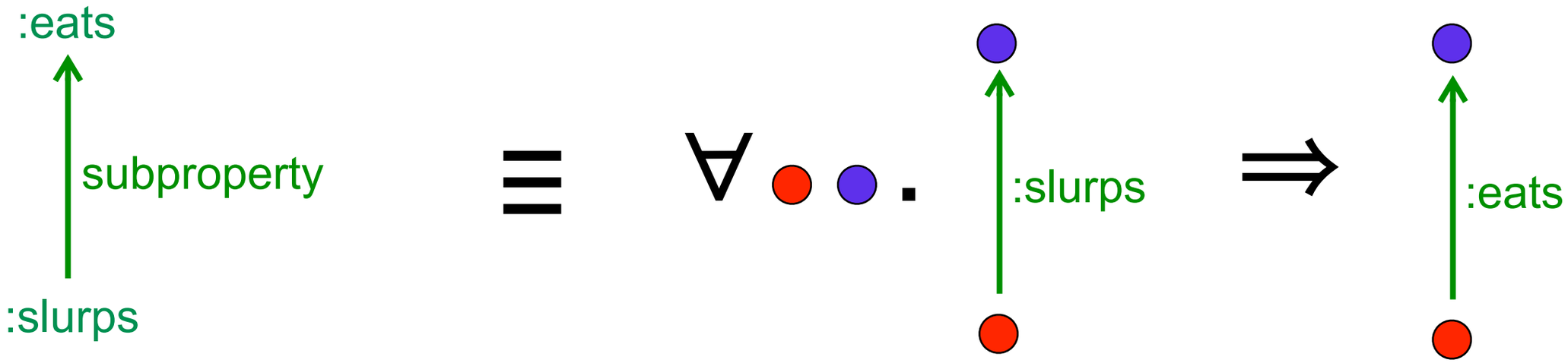

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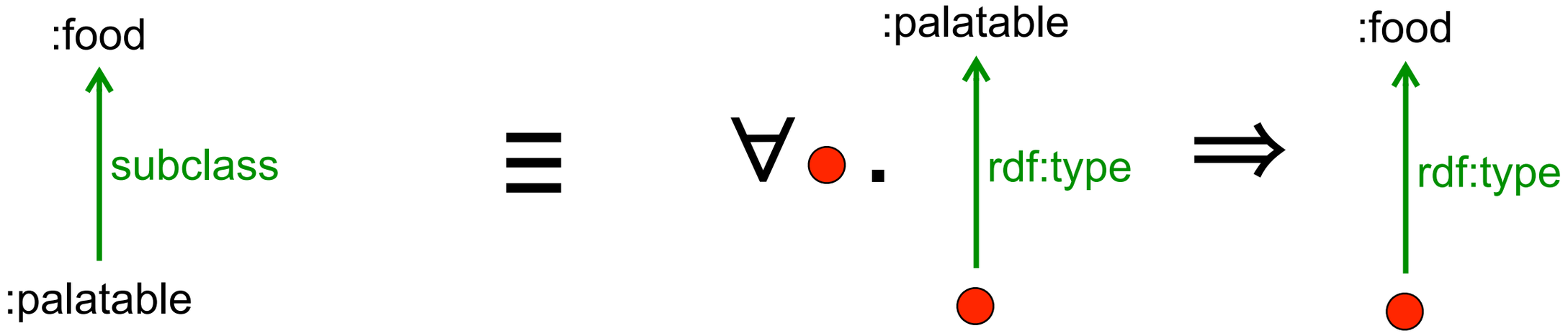

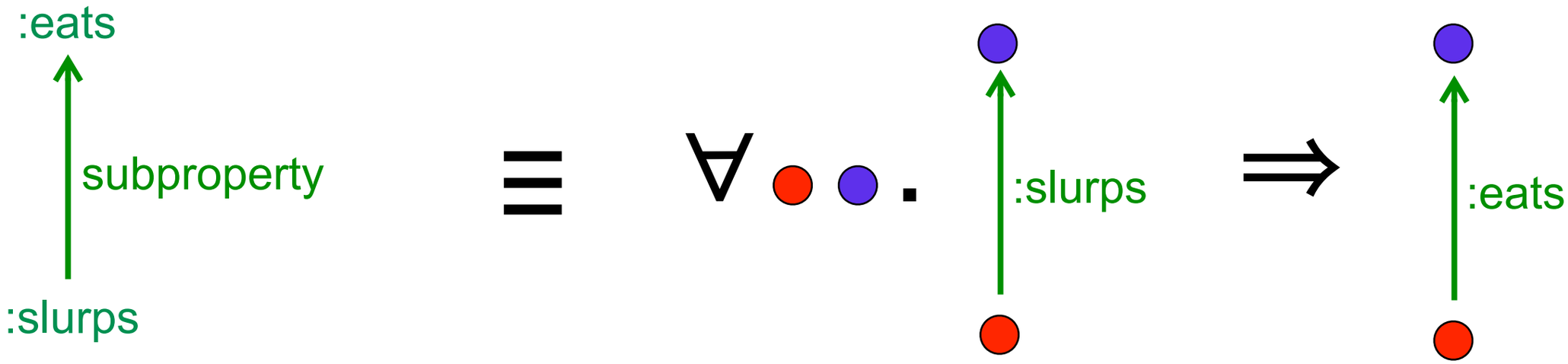

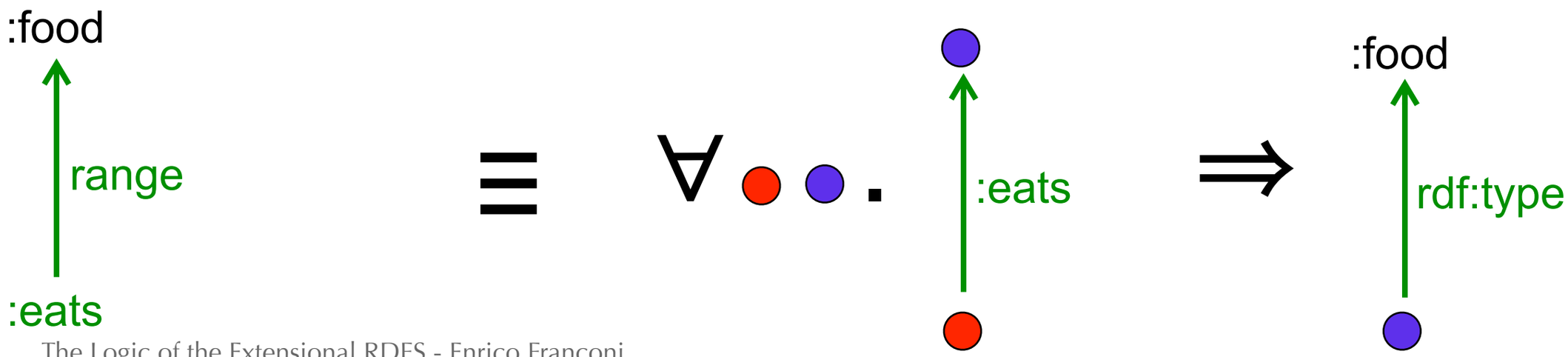

Example of RDFS entailment

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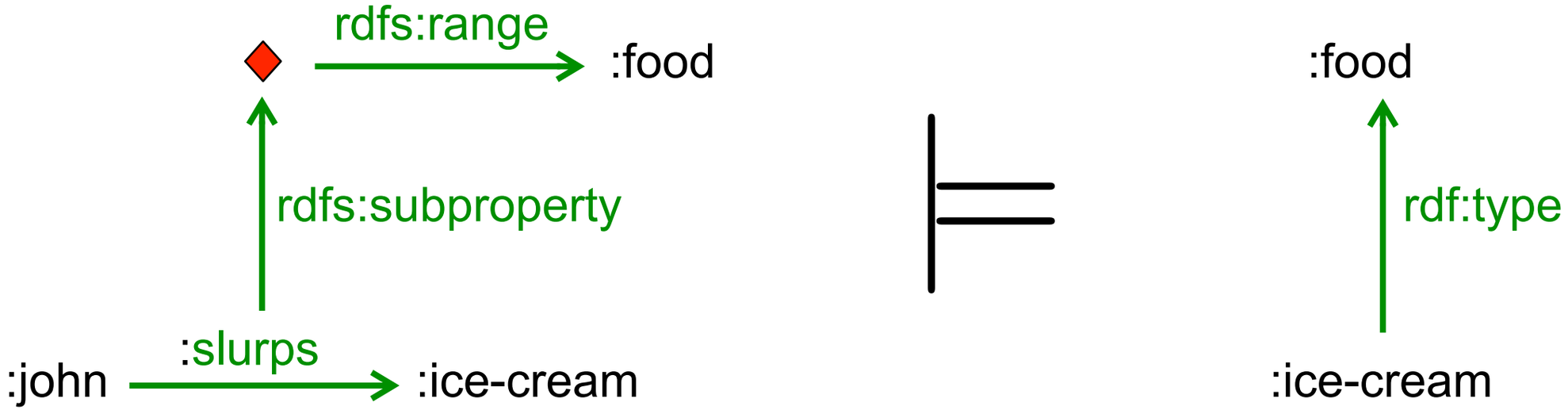

Real meaning of RDFS vocabulary

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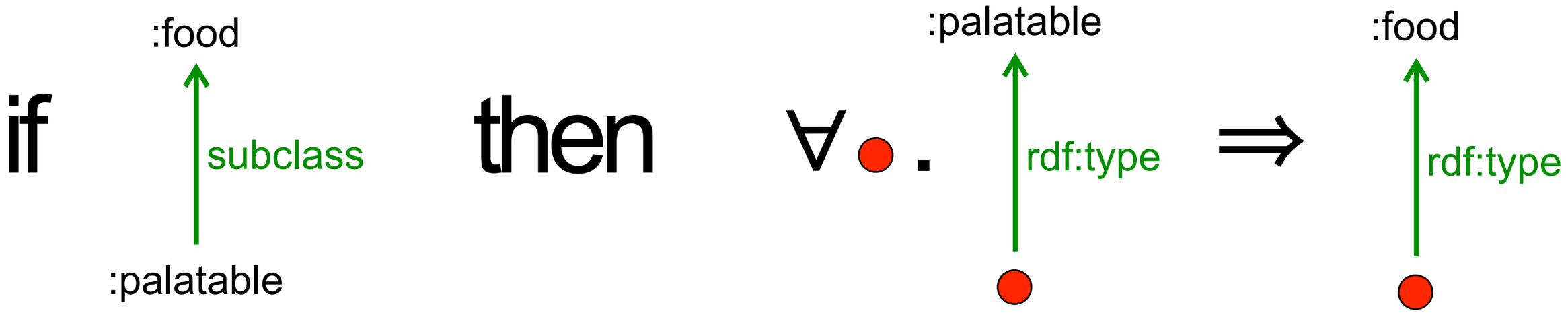

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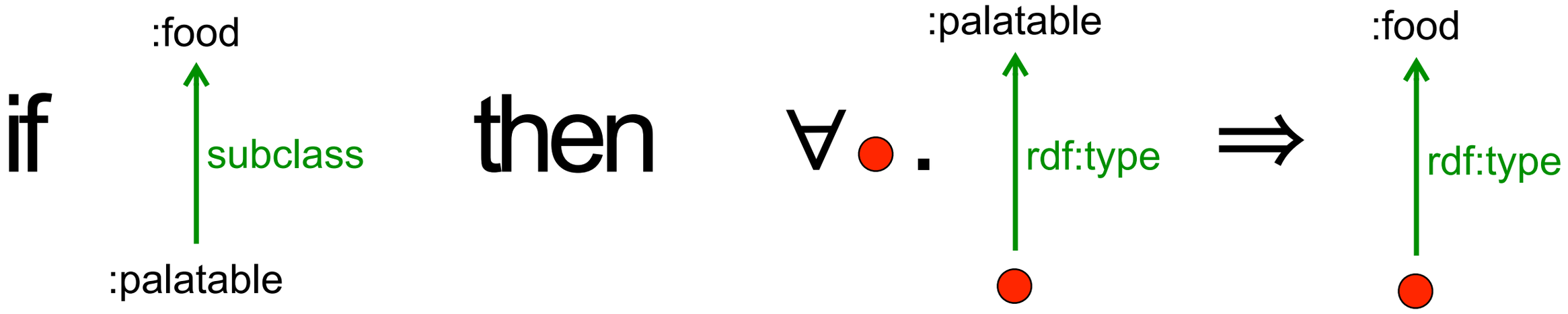

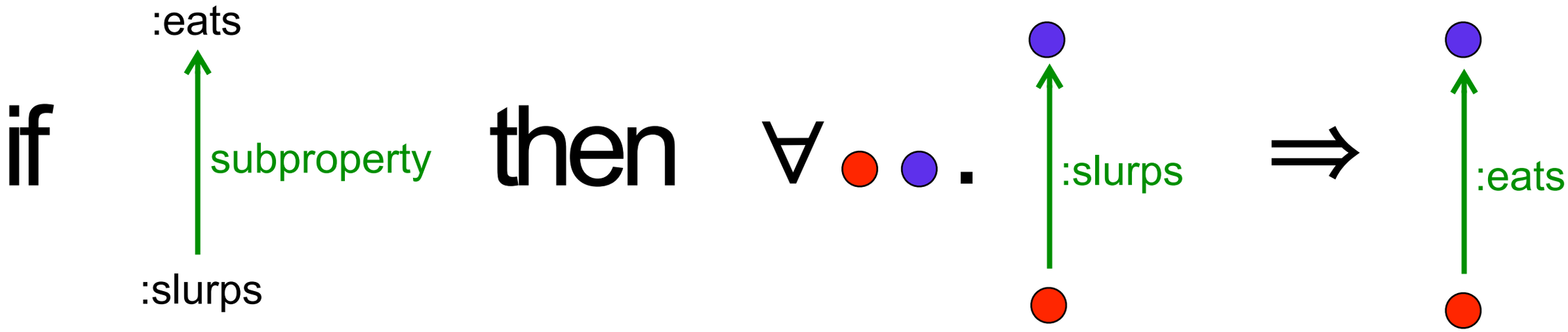

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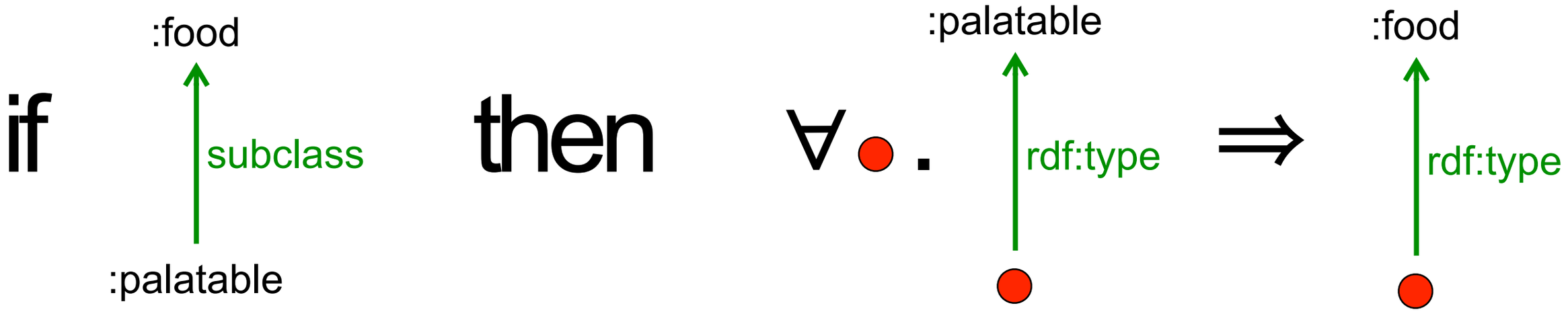
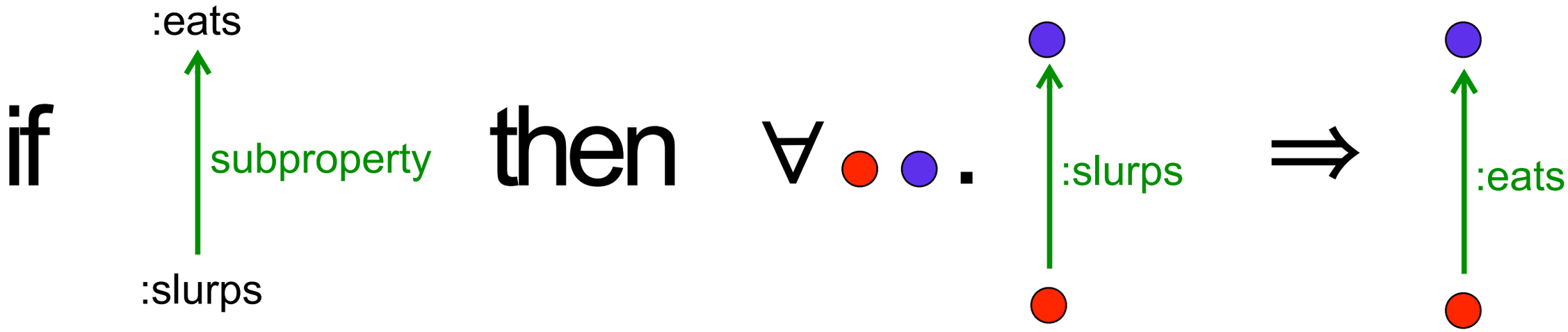
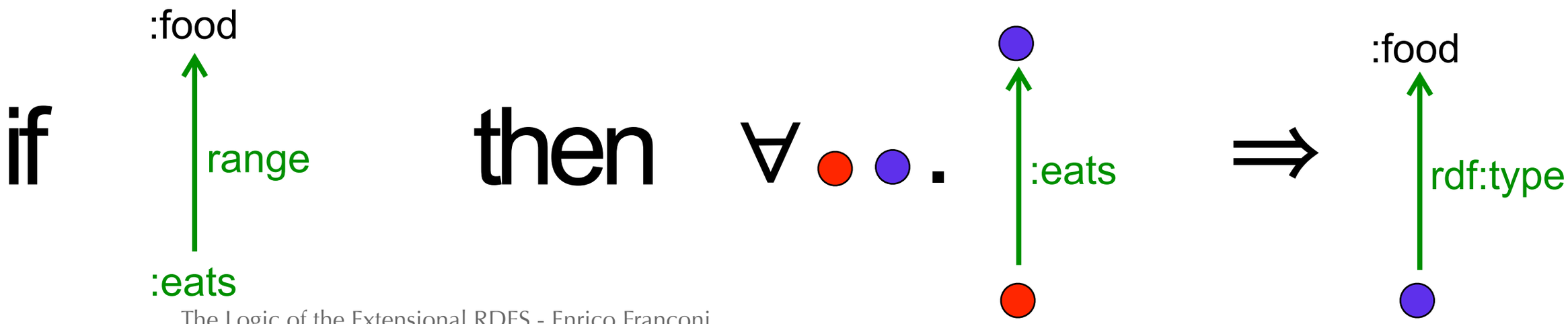






Intensional semantics of RDFS

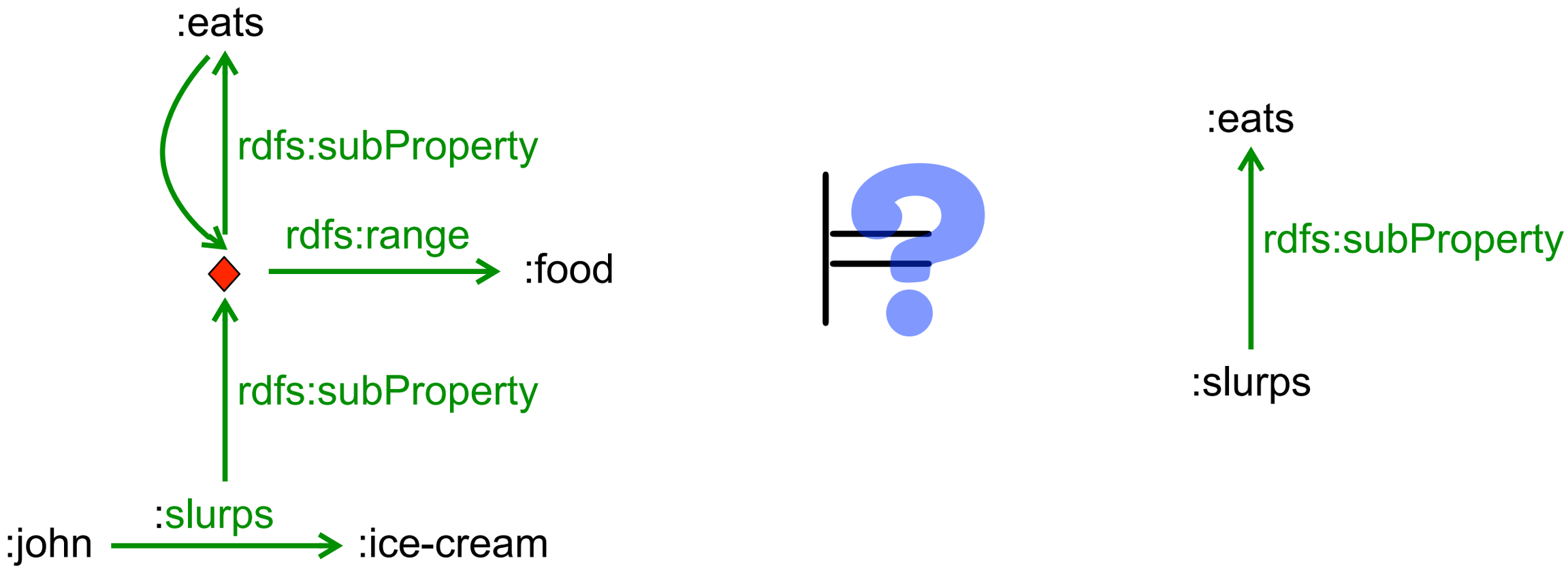
$$\forall a, b.\ T(a, \text{rdfs:subclass}, b) \longrightarrow \forall x.\ T(x, \text{rdf:type}, a) \rightarrow T(x, \text{rdf:type}, b)$$

$$\forall a, b.\ T(a, \text{rdfs:subproperty}, b) \longrightarrow \forall x, y.\ T(x, a, y) \rightarrow T(x, b, y)$$

$$\forall a, c.\ T(a, \text{rdfs:domain}, c) \longrightarrow \forall x, y.\ T(x, a, y) \rightarrow T(x, \text{rdf:type}, c)$$

$$\forall a, d.\ T(a, \text{rdfs:range}, d) \longrightarrow \forall x, y.\ T(x, a, y) \rightarrow T(y, \text{rdf:type}, d)$$

Example revisited ☹



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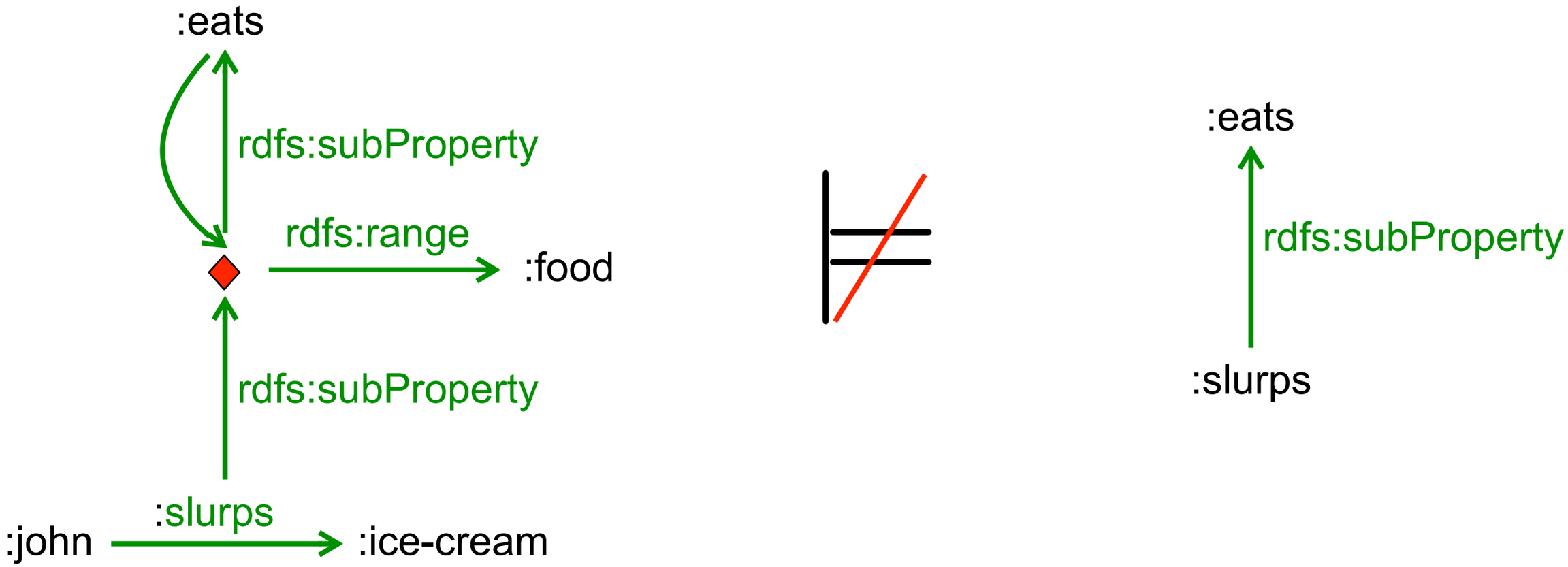

Normative semantics of RDFS

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$$\forall a, b.\ T(a, \text{rdfs:subclass}, b) \longrightarrow C(a) \wedge C(b) \wedge \forall x.\ T(x, \text{rdf:type}, a) \rightarrow T(x, \text{rdf:type}, b)$$

$$\forall a, b.\ T(a, \text{rdfs:subproperty}, b) \longrightarrow P(a) \wedge P(b) \wedge \forall x, y.\ T(x, a, y) \rightarrow T(x, b, y)$$

$$\forall a, c.\ T(a, \text{rdfs:domain}, c) \longrightarrow \forall x, y.\ T(x, a, y) \rightarrow T(x, \text{rdf:type}, c)$$

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$$\forall a, d.\ T(a, \text{rdfs:range}, d) \longrightarrow \forall x, y.\ T(x, a, y) \rightarrow T(y, \text{rdf:type}, d)$$

$$\forall a, b, c.\ T(a, \text{rdfs:subclass}, b) \wedge T(b, \text{rdfs:subclass}, c) \longrightarrow T(a, \text{rdfs:subclass}, c)$$
$$\forall a.\ C(a) \longrightarrow T(a, \text{rdfs:subclass}, a)$$
$$\forall a, b, c.\ T(a, \text{rdfs:subproperty}, b) \wedge T(b, \text{rdfs:subproperty}, c) \longrightarrow T(a, \text{rdfs:subproperty}, c)$$
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Normative semantics of RDFS

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$$\forall a.\ P(a) \longrightarrow T(a, \text{rdfs:subproperty}, a)$$

$$\forall a, b.\ (a, \text{rdfs:domain}, b) \longrightarrow P(a) \wedge C(b)$$
$$\forall a, b.\ (a, \text{rdfs:range}, b) \longrightarrow P(a) \wedge C(b)$$
$$\forall a, b.\ (a, \text{rdf:type}, b) \longrightarrow C(b)$$
$$\forall a, b, c.\ (a, b, c) \longrightarrow P(b)$$
$$P(\text{rdfs:subclass}) \wedge P(\text{rdfs:subproperty}) \wedge P(\text{rdfs:domain}) \wedge P(\text{rdfs:range}) \wedge P(\text{rdf:type})$$

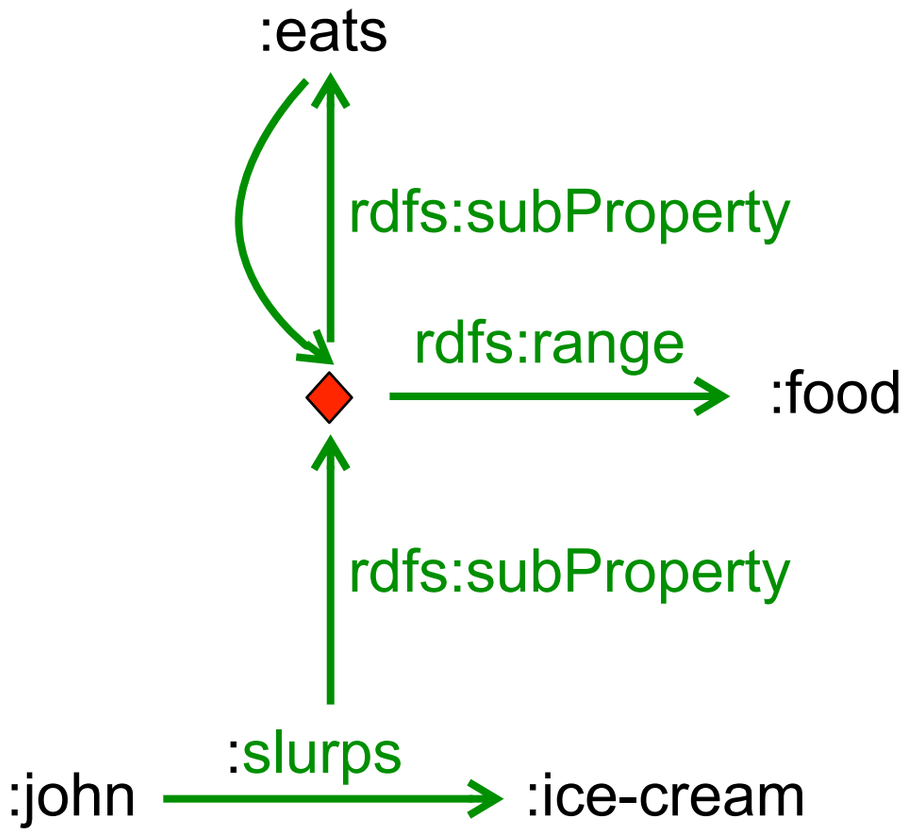
Entailment in normative RDFS

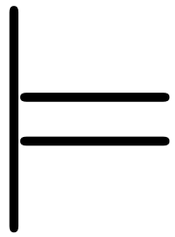
$$G \models_{\mathrm{RDFS}} H$$

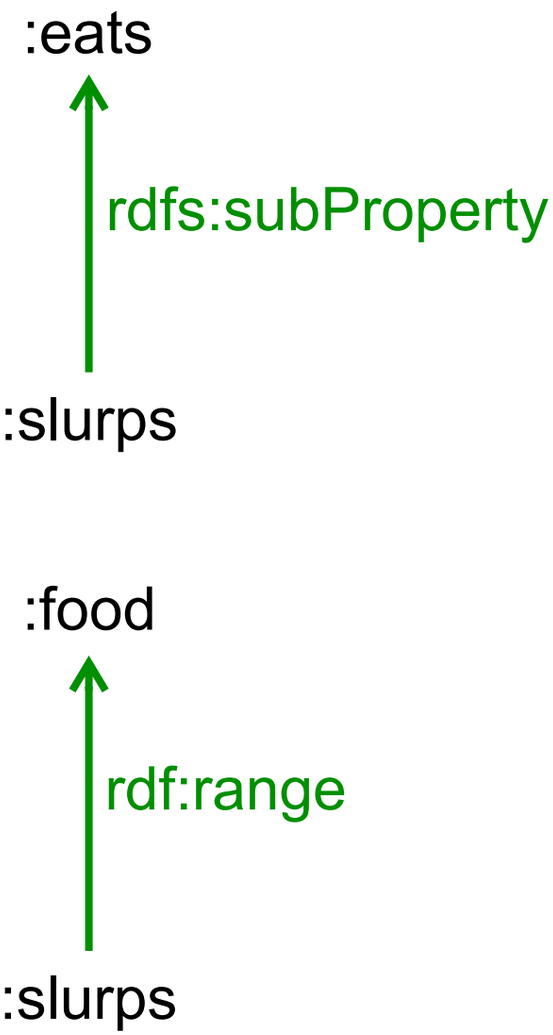
- [ter Horst, 2005] NP-complete in the size of the graphs
  - Polynomial if H does not contain bnodes

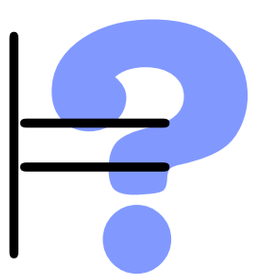
- Algorithm: reduction to RDF entailment through a polynomial completion of graph G

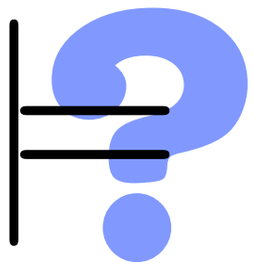
Example revisited ☹

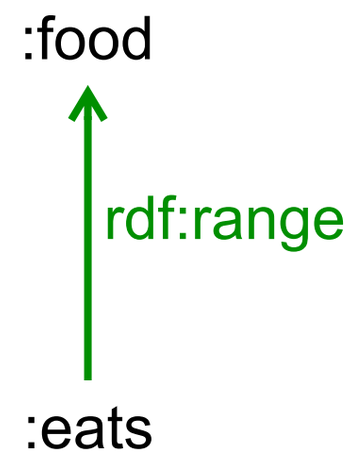












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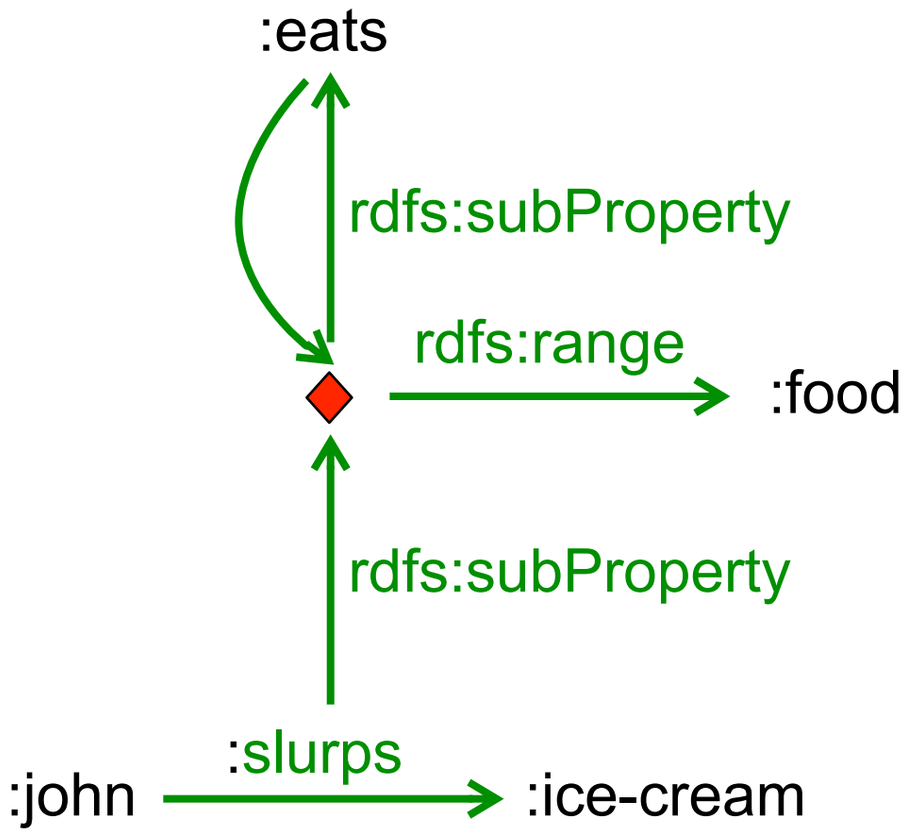

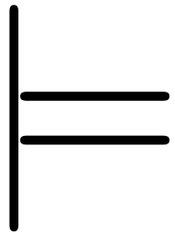

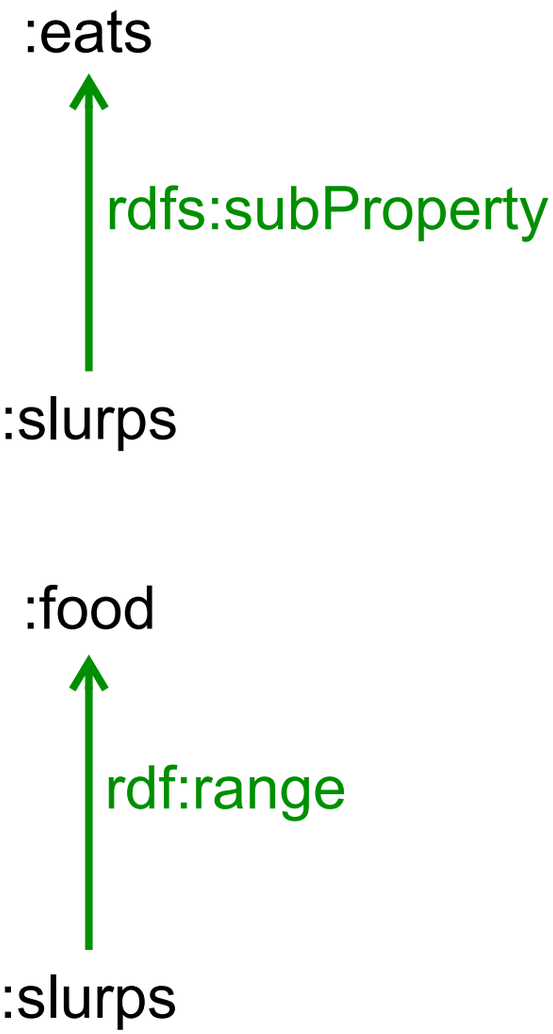

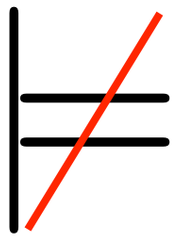

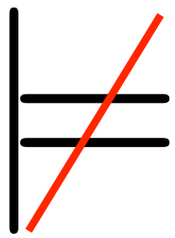

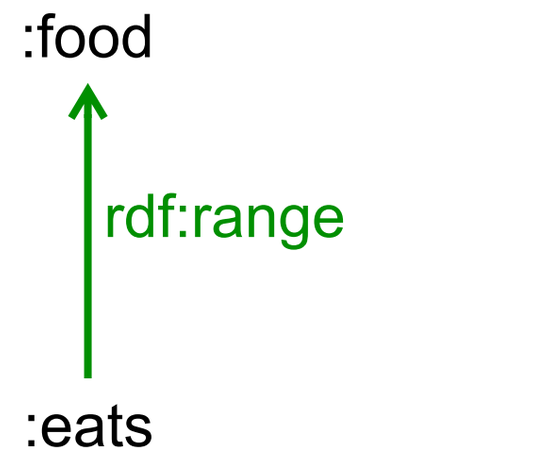

Extensional semantics of RDFS

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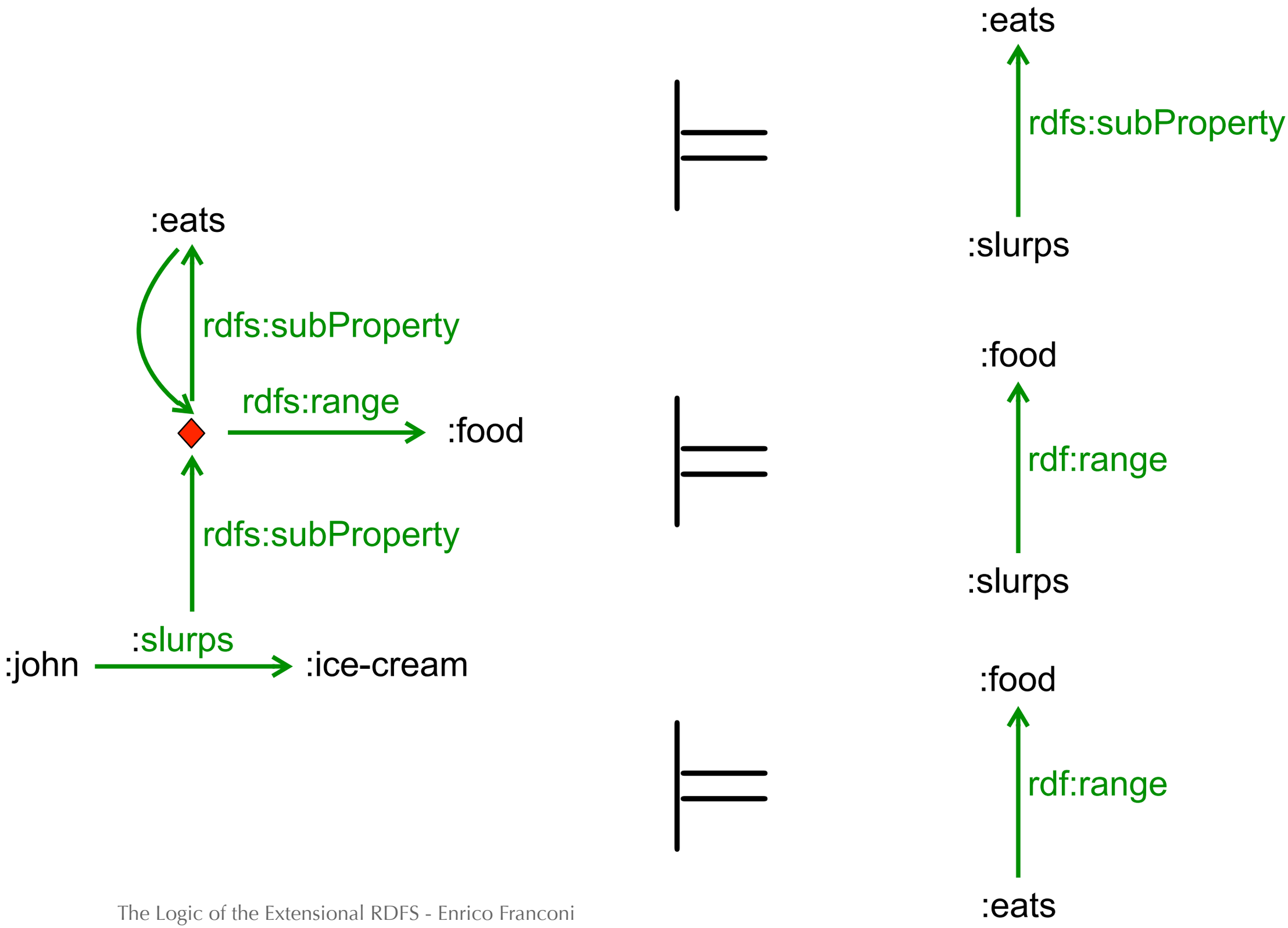


Main result: Entailment in extensional RDFS

$$G \models_{\mathrm{RDFS}^e} H$$

- NP-complete in the size of the graphs
  - Polynomial if H does not contain bnodes

- Sound and complete proof system: reduction to RDF entailment through a carefully designed polynomial completion of graph G

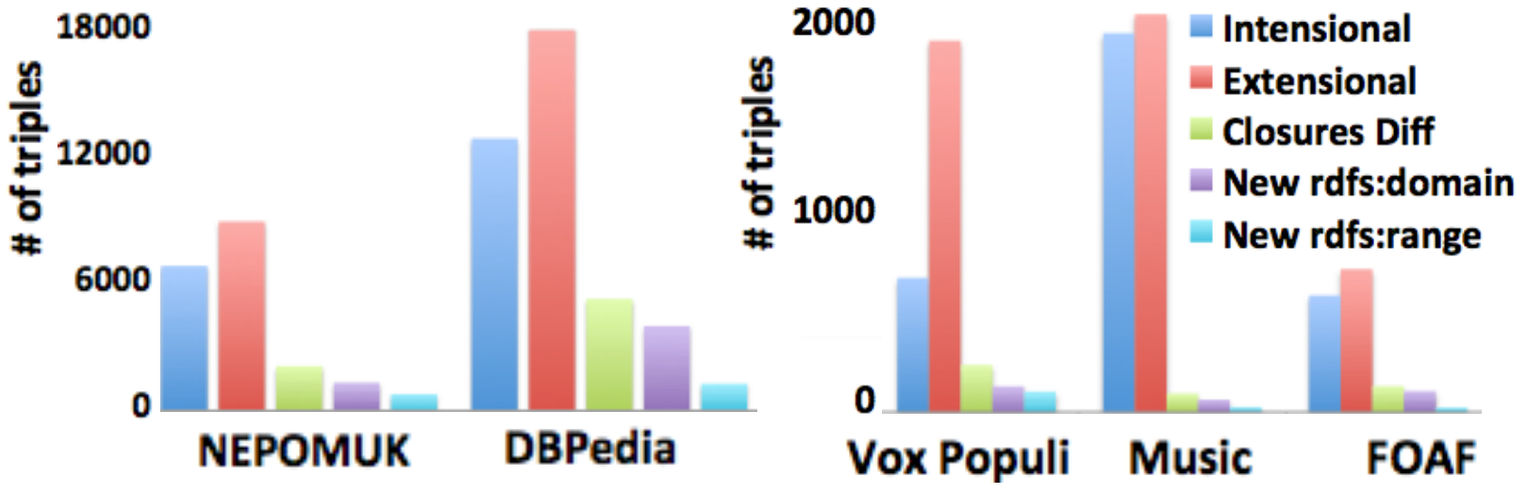
  (forward-chaining rule system)

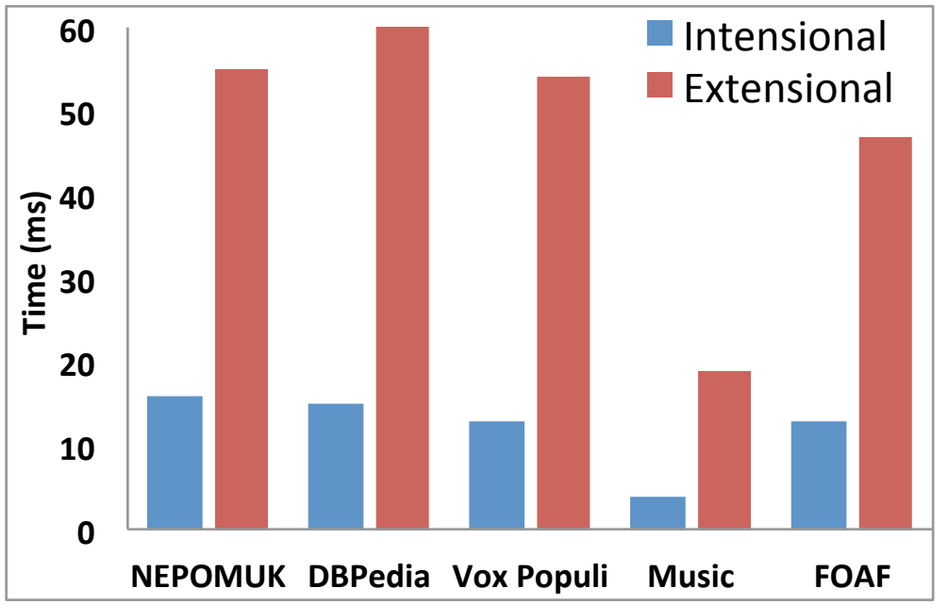
Expressivity of extensional RDFS

- In normative RDFS:
  - rdfs:subclass ≠ owl:subclass
  - rdfs:subproperty ≠ owl:subproperty
  - rdfs:domain ≠ owl:domain
  - rdfs:range ≠ owl:range
- but in extensional RDFS:
  - rdfs+:subclass = owl:subclass
  - rdfs+:subproperty = owl:subproperty
  - rdfs+:domain = owl:domain
  - rdfs+:range = owl:range

Evaluation

- A naîve simple extension of Apache Jena





Conclusions

- Proper meaningful extensional RDFS
  - fully compatible with OWL semantics
  - can be adopted today as standard
  - with full backward compatibility
  - without hampering any optimised reasoner.