Datalog$^{\pm}$: A Unifying Framework for Ontological Reasoning and Query Answering

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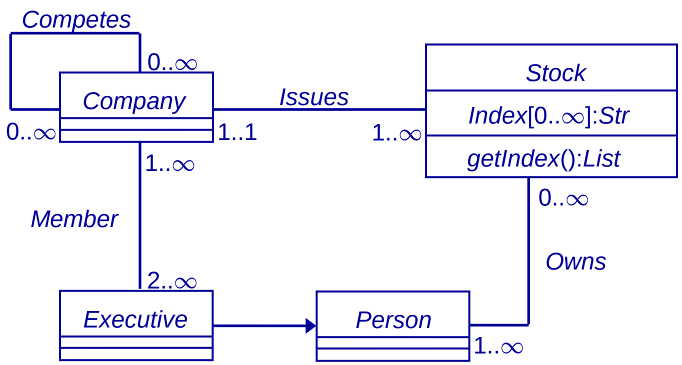
The Need for Querying Ontologies



UML Class Diagrams

student::person

person[age $*\!\Rightarrow$ number]

person[age {0:1} $*\!\Rightarrow$ number]

person[name {1:$*$} $*\!\Rightarrow$ string]

F-Logic

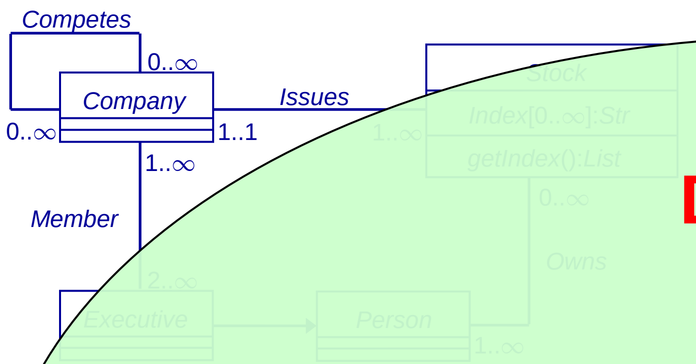
$$
\begin{aligned}
\mathit{Speaker} &\sqsubseteq \mathit{Person} \sqcap \exists \mathit{gives}.\mathit{Talk} \\
\mathit{Tutorial} &\sqsubseteq \forall \mathit{attendedBy}.\mathit{Student} \\
\mathit{attendedBy} &\sqsubseteq \mathit{attended}^{-1} \\
\mathit{Speaker} \sqcap \mathit{Student} &\sqsubseteq \bot
\end{aligned}
$$

Description Logics

```
PREFIX abc: ⟨http://example.com/exampleOntology#⟩
SELECT ?capital ?country
WHERE {
    ?x abc:cityname ?capital ;
    ?x abc:isCapitalOf ?y .
    ?x abc:countryname ?country ;
    ?x abc:isInContinent abc:Africa . }
```

SPARQL (with reasoning capabalities)

A Unifying Framework for Ontology Querying



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person[age *⇒ number]
person[age {0:1} *⇒ number]
person[name {1:*} *⇒ string]

F-Logic

Datalog±

- Logical foundations, semantics and decidability
- Complexity results for ontological query answering
- Identification of tractable fragments

PREFIX abc: ⟨http://example.com/exampleOntology#⟩
SELECT ?capital ?country
WHERE {
?x abc:cityname ?capital ;
?y abc:isCapitalOf ?y .
?x abc:countryname ?country ;
?x abc:isInContinent abc:Africa . }

Tutorial ⊑ ∀attendedBy.Student
attendedBy ⊑ attended^{-1}
Speaker ⊓ Student ⊑ ⊥

Description Logics

SPARQL (with reasoning capabalities)

The Datalog$^{\pm}$ Family

- Datalog: $\forall X \forall Y$ *fatherOf*(X,Y) $\wedge$ *person*(Y) $\rightarrow$ *person*(X)

- Extend Datalog by allowing in the head:

  Existential quantification (TGDs) – $\forall X$ *person*(X) $\rightarrow \exists Y$ *fatherOf*(Y,X) $\wedge$ *person*(Y)

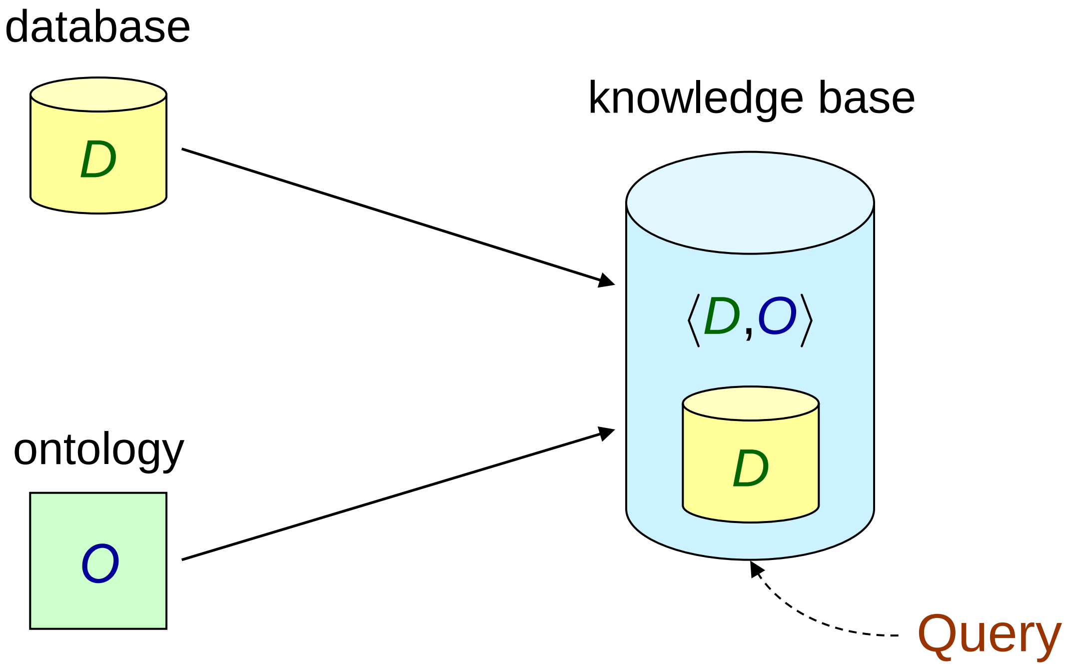
  Equality predicate (EGDs) – $\forall X \forall Y \forall Z$ *fatherOf*(Y,X) $\wedge$ *fatherOf*(Z,X) $\rightarrow$ Y = Z

  Constant false (Negative Constraints) – $\forall X \forall Y$ *fatherOf*(Y,X) $\wedge$ *motherOf*(Y,X) $\rightarrow \perp$

- But query answering under Datalog[$\exists$] is already undecidable
  see, e.g., [Beeri & Vardi, ICALP 1981] and [Calì, G. & Kifer, KR 2008]

- Datalog[$\exists$,=,$\perp$] is syntactically restricted $\rightarrow$ **Datalog$^{\pm}$**

Ontological Query Answering



Ontological Query Answering: Example

D = *person*(john)

O =

$\forall P\ \textit{person}(P) \rightarrow \exists F\ \textit{fatherOf}(F,P)$

$\forall F \forall P\ \textit{fatherOf}(F,P) \rightarrow \textit{person}(F)$

each model of $\langle D, O\rangle$ $\supseteq$ …*fatherOf*(*z*,john) *person*(*z*)…

$\exists X\ \textit{fatherOf}(X,\text{john}) \wedge \textit{person}(X)$ ✓

$\exists X\ \textit{fatherOf}(\text{john},X)$ ✗

Decidable Languages

- Guarded Datalog[∃]

- Linear Datalog[∃]

- Sticky Datalog[∃]

Guarded Datalog[$\exists$]

- All $\forall$-variables occur in one body atom – guard atom

$$\forall E \forall D\ \mathit{employee}(E) \wedge \mathit{managerOf}(E,D) \rightarrow \exists F\ \mathit{supervisorOf}(E,F)$$

guard

- Models of finite treewidth $\Rightarrow$ decidability of query answering

  [Calì, G. & Kifer, KR 2008, JAIR 2013] related to [Andréka, Németi & van Benthem, J. Philosophical Logic 1998] and [Grädel, J. Symb. Log. 1999]

- Query answering is PTIME-complete in data complexity

[Calì, G. & Kifer, KR 2008, JAIR 2013]

Linear Datalog[$\exists$]

- Rules with just one body-atom

$$\forall E\ \mathit{supervisorOf}(E,E) \rightarrow \exists D\ \mathit{managerOf}(E,D)$$

- Query answering is first-order rewritable

- Same data complexity as plain SQL queries (in AC_0)

[Calì, G. & Lukasiewicz, J. Web. Sem. 2012]

Sticky Datalog[∃]

- Join-variables "stick" to the inferred atoms

D is the director of P, and (s)he is an expert in A

$$\forall P \forall N \forall A\ \mathit{project}(P,N) \wedge \mathit{inArea}(P,A) \rightarrow \exists D\ \mathit{directorOf}(D,P,A)$$

N is the name of project P

- Query answering is first-order rewritable

- Same data complexity as plain SQL queries (in AC_0)

[Calì, G. & Pieris, Artif. Intell. 2012]

Datalog[$\exists$,=,$\perp$]

- Every decidable Datalog[$\exists$] language can be enriched with:
  - Non-Conflicting EGDs – no interaction with TGDs

    $\forall X \forall Y \forall Z\ \mathit{fatherOf}(Y,X) \wedge \mathit{fatherOf}(Z,X) \rightarrow Y = Z$

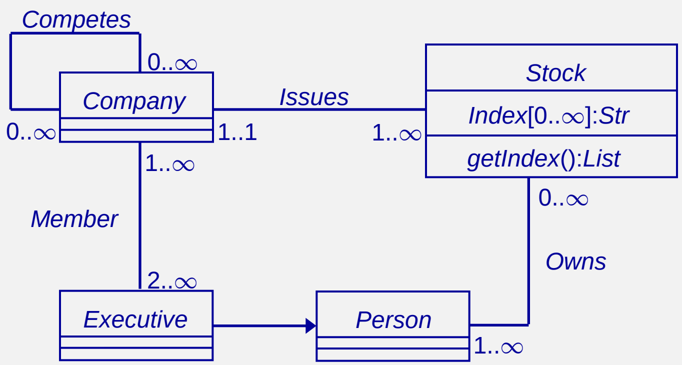
  - Negative constraints

    $\forall X \forall Y\ \mathit{fatherOf}(Y,X) \wedge \mathit{motherOf}(Y,X) \rightarrow \perp$

- The complexity of query answering remains the same

see, e.g., [Calì, G. & Pieris, Artif. Intell. 2012]

A Unifying Framework for Ontology Querying



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SPARQL (with reasoning capabalities)

Querying UML Class Diagrams: Example

Executive(john)

Member(john,LU)

Stock(BAY)

Issues(BA,BAY)

Owns(john,BAY)

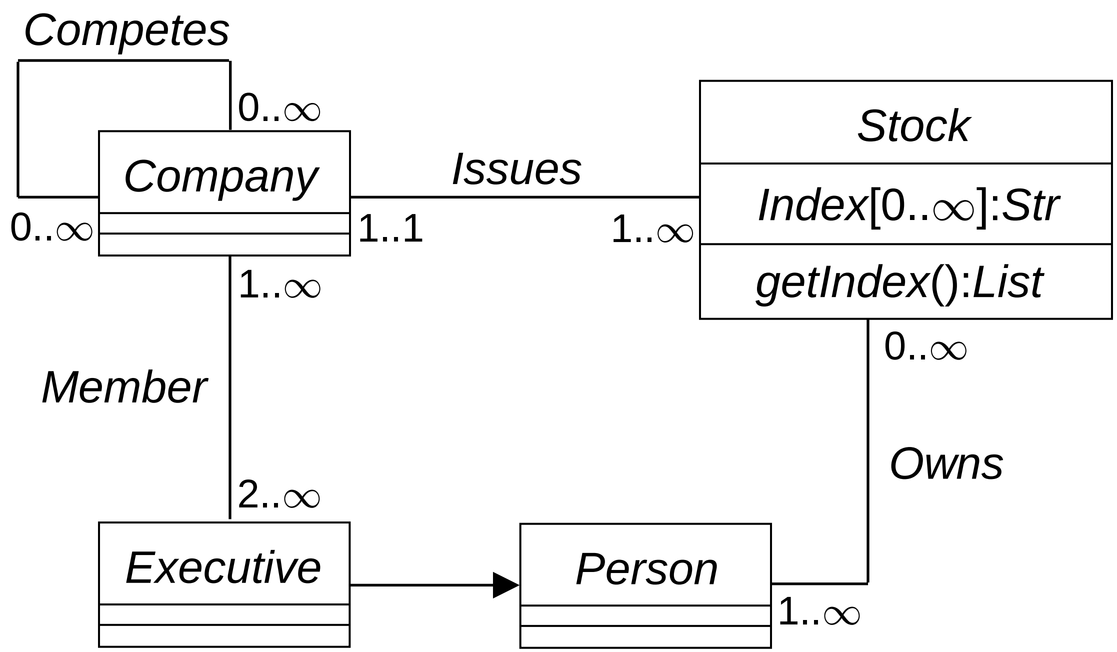
Competes(LU,BA)

***Person*(john)**

***Company*(LU)**

***Company*(BA)**

+



Does anybody have a potential conflict of interest?

$\mathit{Conflict} \leftarrow \mathit{Person}(P), \mathit{Company}(C_1), \mathit{Company}(C_2), \mathit{Stock}(S),$
$\mathit{Owns}(P,S), \mathit{Member}(P,C_1), \mathit{Issues}(C_2,S), \mathit{Competes}(C_1,C_2)$ ✓

Querying UML Class Diagrams: Example

Executive(john)

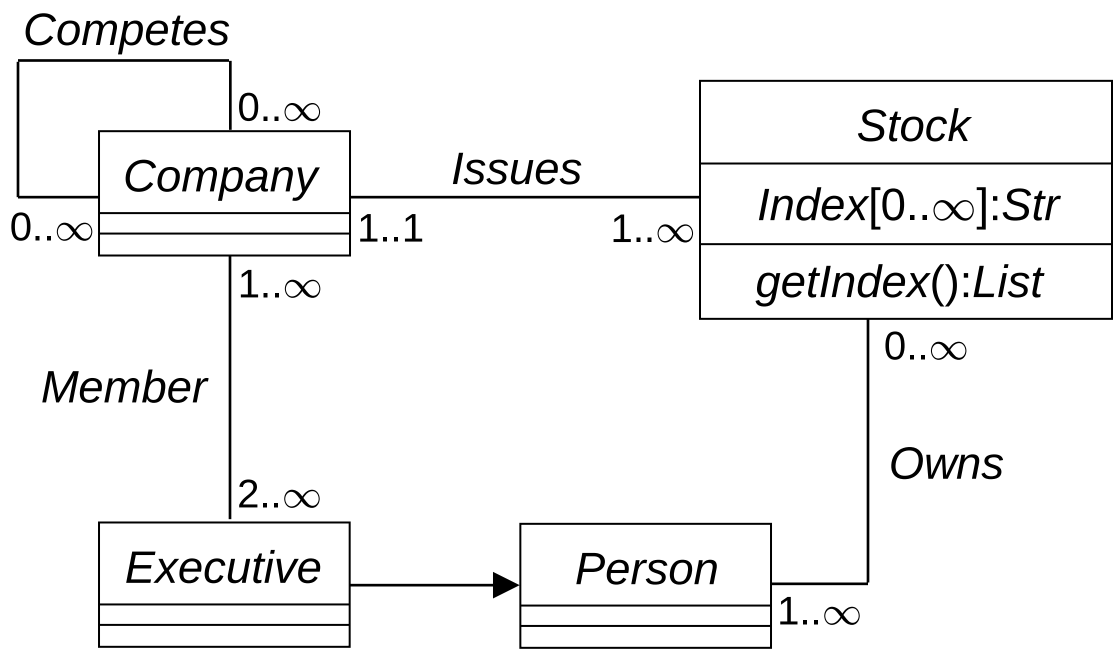
Member(john,LU)

Stock(BAY)

Issues(BA,BAY)

Owns(john,BAY)

Competes(LU,BA)



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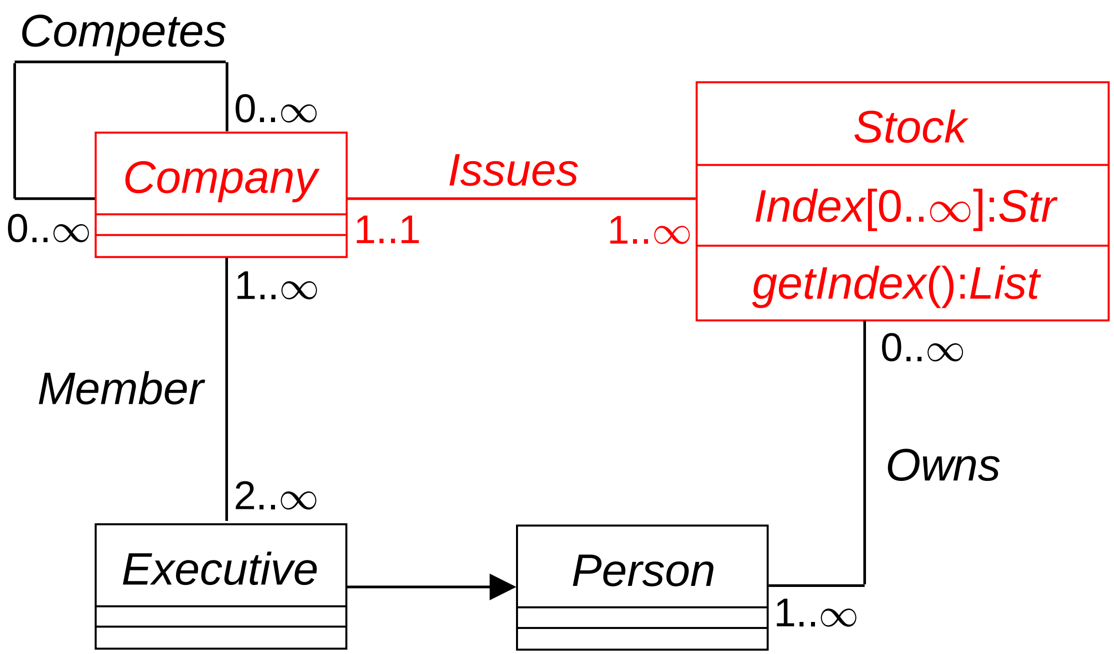
From Diagrams to First-Order Logic (Datalog$^{\pm}$)



$\forall X\ \mathit{Company}(X) \rightarrow \exists Y\ \mathit{Issues}(X,Y)$

$\forall X\ \mathit{Stock}(X) \rightarrow \exists Y\ \mathit{Issues}(Y,X)$

$\forall X \forall Y \forall Z\ \mathit{Stock}(X) \wedge \mathit{Issues}(Y,X) \wedge \mathit{Issues}(Z,X) \rightarrow Y = Z$

$\forall X \forall Y\ \mathit{Stock}(X) \wedge \mathit{Index}(X,Y) \rightarrow \mathit{Str}(Y)$

$\forall X \forall Y\ \mathit{Stock}(X) \wedge \mathit{getIndex}(X,Y) \rightarrow \mathit{List}(Y)$

[Berardi et al., Artif. Intell. 2005]

Lean UML Class Diagrams as Guarded Rules

$\forall X \forall Y\ C(X) \wedge Attr(X,Y) \rightarrow T(Y)$

$\forall X\ C(X) \rightarrow \exists Y_1 \ldots \exists Y_n\ Attr(X,Y_1) \wedge \ldots \wedge Attr(X,Y_n)$

$\forall X\ C_1(X) \rightarrow C_2(X)$

classes

$\forall X_1 \forall X_2\ A(X_1,X_2) \rightarrow C_1(X_1) \wedge C_2(X_2)$

$\forall X_1 \forall X_2 \forall Y\ A(X_1,X_2) \wedge glue(X_1,X_2,Y) \rightarrow C_A(Y)$

$\forall X_1 \forall X_2\ A(X_1,X_2) \rightarrow \exists Y\ glue(X_1,X_2,Y)$

$\forall X\ C(X) \rightarrow \exists Y_1 \ldots \exists Y_n\ A(X,Y_1) \wedge \ldots \wedge A(X,Y_n)$

$\forall X\ C(X) \rightarrow \exists Y_1 \ldots \exists Y_n\ A(Y_1,X) \wedge \ldots \wedge A(Y_n,X)$

$\forall X_1 \forall X_2\ A_1(X_1,X_2) \rightarrow A_2(X_1,X_2)$

associations

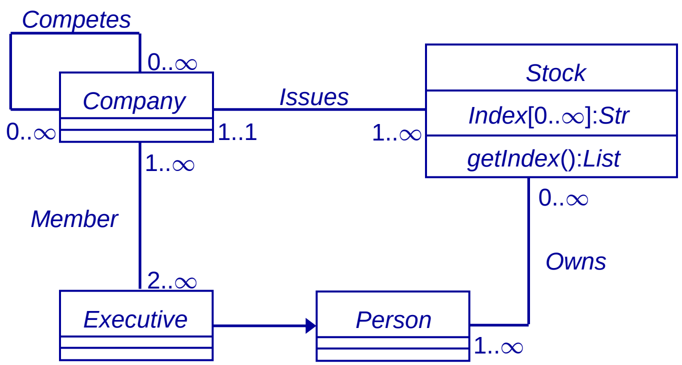
$\forall X\ C_1(X) \wedge \ldots \wedge C_n(X) \rightarrow C(X)$

$\forall X \forall Y\ C(X) \wedge Attr(Y,X) \rightarrow T(Y)$

additional constraints

[Calì, G., Orsi & Pieris, FOSSACS 2012]

A Unifying Framework for Ontology Querying



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SPARQL (with reasoning capabalities)

The DL-Lite Family

Popular family of DLs with AC_0 data complexity

DL-Lite TBox	First-Order Representation (Datalog$^{\pm}$)
DL-Lite$_{core}$	
$professor \sqsubseteq \exists teachesTo$	$\forall X\ professor(X) \rightarrow \exists Y\ teachesTo(X,Y)$
$professor \sqsubseteq \neg student$	$\forall X\ professor(X) \wedge student(X) \rightarrow \bot$
DL-Lite$_R$ (OWL 2 QL)	
$hasTutor^{-} \sqsubseteq teachesTo$	$\forall X \forall Y\ hasTutor(X,Y) \rightarrow teachesTo(Y,X)$
DL-Lite$_F$	
$\mathrm{funct}(hasTutor)$	$\forall X \forall Y \forall Z\ hasTutor(X,Y) \wedge hasTutor(X,Z) \rightarrow Y = Z$

[Calvanese, De Giacomo, Lembo, Lenzerini & Rosati, J. Autom. Reasoning 2007]

The $\mathcal{EL}$ Family

Popular family of DLs with PTIME data complexity (OWL 2 EL)

$\mathcal{ELHI}^{\neg}$ TBox	First-Order Representation (Datalog$^{\pm}$)
$A \sqcap B \sqsubseteq C$	$\forall X \; A(X) \wedge B(X) \rightarrow C(X)$
$A \sqsubseteq \exists R$	$\forall X \; A(X) \rightarrow \exists Y \; R(X,Y)$
$A \sqsubseteq \exists R.B$	$\forall X \; A(X) \rightarrow \exists Y \; R(X,Y) \wedge B(Y)$
$\exists R \sqsubseteq A$	$\forall X \forall Y \; R(X,Y) \rightarrow A(X)$
$\exists R.A \sqsubseteq B$	$\forall X \forall Y \; R(X,Y) \wedge A(X) \rightarrow B(X)$
$A \sqsubseteq \neg B$	$\forall X \; A(X) \wedge B(X) \rightarrow \bot$
$R \sqsubseteq \neg P$	$\forall X \forall Y \; R(X,Y) \wedge P(X,Y) \rightarrow \bot$

see, e.g., [Baader, IJCAI 2003], [Rosati, DL 2007] and [Pérez-Urbina et al., J. Applied Logic 2010]

DLs vs. Datalog[$\exists$,=,$\perp$]

- **Theorem:** For query answering,

$$\text{DL-Lite} \quad \leq_L \quad \text{Linear and Sticky}$$

$$\mathcal{ELHI}^{\neg} \quad \leq_L \quad \text{Guarded}$$

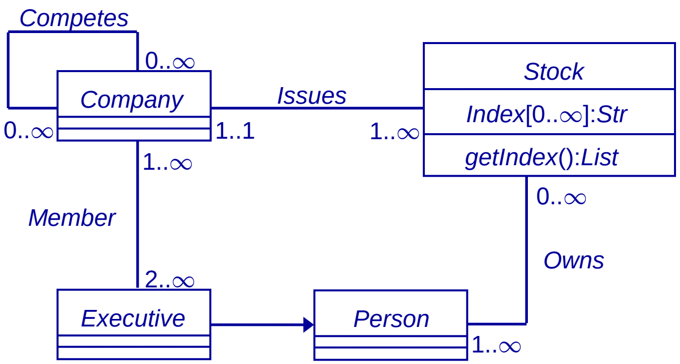
- **Theorem**: Guarded/Linear/Sticky are strictly more expressive

$$\forall E\ \mathit{ceo}(E) \rightarrow \mathit{managerOf}(E,E)$$

$$\forall E\ \mathit{supervisorOf}(E,E) \rightarrow \exists D\ \mathit{managerOf}(E,D)$$

- **Remark:** the data complexity is preserved

A Unifying Framework for Ontology Querying



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```

SPARQL (with reasoning capabalities)

From F-Logic Lite to First-Order Logic (Datalog$^{\pm}$)

$type(O,A,T) \wedge data(O,A,V) \rightarrow member(V,T)$

$sub(C_1,C_3) \wedge sub(C_3,C_2) \rightarrow sub(C_1,C_2)$

$member(O,C) \wedge sub(C,C_1) \rightarrow member(O,C_1)$

$data(O,A,V) \wedge data(O,A,W), funct(A,O) \rightarrow V = W$

$mandatory(A,O) \rightarrow \exists V\, data(O,A,V)$

$member(O,C) \wedge type(C,A,T) \rightarrow type(O,A,T)$

$sub(C,C_1) \wedge type(C_1,A,T) \rightarrow type(C,A,T)$

$type(C,A,T_1) \wedge sub(T_1,T) \rightarrow type(C,A,T)$

$sub(C,C_1) \wedge mandatory(A,C_1) \rightarrow mandatory(A,C)$

$member(O,C) \wedge mandatory(A,C) \rightarrow member(A,O)$

$sub(C,C_1) \wedge funct(A,C_1) \rightarrow funct(A,C)$

$member(O,C) \wedge funct(A,C) \rightarrow funct(A,O)$

[Calì & Kifer, VLDB 2006]

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$member(O,C) \wedge sub(C,C_1) \rightarrow member(O,C_1)$

$\boxed{data(O,A,V) \wedge data(O,A,W), funct(A,O) \rightarrow V = W}$ Non-conflicting EGD

$mandatory(A,O) \rightarrow \exists V\, data(O,A,V)$

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[Calì & Kifer, VLDB 2006]

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$\boxed{type(O,A,T) \wedge data(O,A,V)} \rightarrow member(V,T)$

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$member(O,C) \wedge sub(C,C_1) \rightarrow member(O,C_1)$

Non-guarded rules!

$mandatory(A,O) \rightarrow \exists V\ data(O,A,V)$

$\boxed{member(O,C) \wedge type(C,A,T)} \rightarrow type(O,A,T)$

$\boxed{sub(C,C_1) \wedge type(C_1,A,T)} \rightarrow type(C,A,T)$

$\boxed{type(C,A,T_1) \wedge sub(T_1,T)} \rightarrow type(C,A,T)$

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$\boxed{sub(C,C_1) \wedge funct(A,C_1)} \rightarrow funct(A,C)$

$\boxed{member(O,C) \wedge funct(A,C)} \rightarrow funct(A,O)$

More expressive

Datalog[∃] language?

[Calì & Kifer, VLDB 2006]

Weakly-Guarded Datalog[∃]

- All ∀-variables at affected positions occur in one body atom

  unify with an ∃-position or an affected position

- Models of finite treewidth ⇒ decidability of query answering

  related to [Andréka, Németi & van Benthem, J. Philosophical Logic 1998] and [Grädel , J. Symb. Log. 1999]

- Query answering is EXPTIME-complete in data complexity – PTIME-complete if the Polynomial Cloud Criterion (PCC) holds

[Calì, G. & Kifer, KR 2008]

F-Logic Lite is Weakly-Guarded

$\mathit{type}(O,A,T) \wedge \mathit{data}(O,A,V) \rightarrow \mathit{member}(V,T)$

$\mathit{sub}(C_1,C_3) \wedge \mathit{sub}(C_3,C_2) \rightarrow \mathit{sub}(C_1,C_2)$

$\mathit{member}(O,C) \wedge \mathit{sub}(C,C_1) \rightarrow \mathit{member}(O,C_1)$

$\mathit{mandatory}(A,O) \rightarrow \exists V\ \mathit{data}(O,A,V)$

$\mathit{member}(O,C) \wedge \mathit{type}(C,A,T) \rightarrow \mathit{type}(O,A,T)$

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$\mathit{sub}(C,C_1) \wedge \mathit{funct}(A,C_1) \rightarrow \mathit{funct}(A,C)$

$\mathit{member}(O,C) \wedge \mathit{funct}(A,C) \rightarrow \mathit{funct}(A,O)$

Affected Positions

data[1]

data[3]

type[1]

member[1]

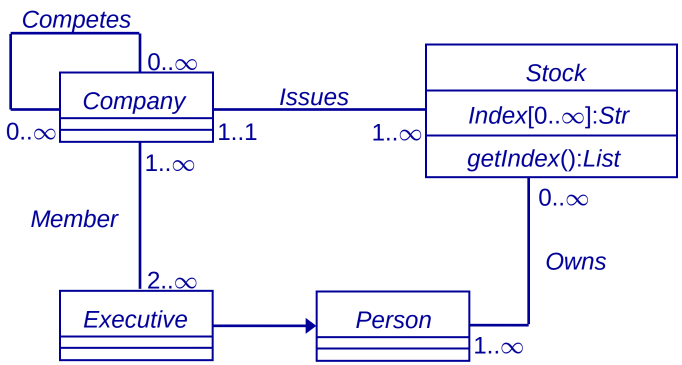
mandatory[2]

funct[2]

the PCC holds

[Calì & Kifer, VLDB 2006]

A Unifying Framework for Ontology Querying



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SPARQL (with reasoning capabalities)

SPARQL Protocol and RDF Query Language

- RDF – data model for representing information in the Web

- … in fact, is a finite set of triples (*subject*, *predicate*, *object*) – or a relational database for the schema {*triple*(.,.,.)}

- SPARQL – the standard language for querying RDF data

Some SPARQL Queries

- P = (?X, name, ?Y) – list of pairs (o_1,o_2) such as o_2 is the name of o_1
- P = (?X, name, B) – list of elements that have a name
- P = (?X, name, ?Y) OPT (?X, phone, ?Y) – for every subject *o*, return *o*, the name of *o*, and the phone number of *o*, if the phone number is available; otherwise, return *o* and its name

From SPARQL to First-Order Logic (Datalog$^{\pm}$)

P = (?X, name, ?Y) – list of pairs (o_1,o_2) such as o_2 is the name of o_1

$$\forall X \forall Y \; \mathit{triple}(X,\mathrm{name},Y) \;\rightarrow\; P(X,Y)$$

From SPARQL to First-Order Logic (Datalog$^{\pm}$)

P = (?X, name, B) – list of elements that have a name

$$\forall X \forall Y \;\; \mathit{triple}(X,\mathrm{name},Y) \rightarrow P(X)$$

From SPARQL to First-Order Logic (Datalog$^{\pm}$)

P = (?X, name, ?Y) OPT (?X, phone, ?Y) – for every subject *o*, return *o*, the name of *o*, and the phone number of *o*, if the phone number is available; otherwise, return *o* and its name

list of individuals with phone number

$\forall X \forall Y\ \mathit{triple}(X,\mathrm{name},Y) \wedge \mathit{triple}(X,\mathrm{phone},Z) \rightarrow P(X,Y,Z) \wedge \mathit{compatible}(Z)$

the third argument (i.e., the phone no.) is missing

$\forall X \forall Y\ \mathit{triple}(X,\mathrm{name},Y) \wedge \neg\mathit{compatible}(X) \rightarrow P'(X,Y)$

From SPARQL to First-Order Logic ($\text{Datalog}^{\pm}$)

P = (?X, name, ?Y) OPT (?X, phone, ?Y) – for every subject *o*, return *o*, the name of *o*, and the phone number of *o*, if the phone number is available; otherwise, return *o* and its name

$$\forall X \forall Y\ \boxed{\mathit{triple}(X,\mathrm{name},Y) \wedge \mathit{triple}(X,\mathrm{phone},Z)} \rightarrow P(X,Y,Z) \wedge \mathit{compatible}(Z)$$

$$\forall X \forall Y\ \mathit{triple}(X,\mathrm{name},Y) \wedge \boxed{\neg \mathit{compatible}(X)} \rightarrow P'(X,Y)$$

Non-guarded rules, and also negation is needed

From SPARQL to First-Order Logic (Datalog$^{\pm}$)

P = (?X, name, ?Y) OPT (?X, phone, ?Y) – for every subject *o*, return *o*, the name of *o*, and the phone number of *o*, if the phone number is available; otherwise, return *o* and its name

$$\forall X \forall Y\ \boxed{triple(X,name,Y) \wedge triple(X,phone,Z)} \rightarrow P(X,Y,Z) \wedge compatible(Z)$$

$$\forall X \forall Y\ triple(X,name,Y) \wedge \boxed{\neg compatible(X)} \rightarrow P'(X,Y)$$

Non-guarded rules, and also negation is needed

Stratified Weakly-Guarded Datalog[∃,¬,⊥]

Additional Functionalities

- Reasoning capabilities – deal with RDFS and OWL vocabularies
- Navigational capabilities – exploit the graph structure of RDF data
- General form of recursion – express natural queries

Theorem: Stratified weakly-guarded Datalog[$\exists,\neg,\perp$] is strictly more expressive than SPARQL enriched with the above functionalities (under the the OWL 2 QL and OWL 2 RL profiles of OWL 2)

Overview

Formalism	**Datalog$^{\pm}$ Language**
Lean UML Class Diagrams	Guarded Datalog[$\exists$,=,$\perp$]
DL-Lite (OWL 2 QL)	Linear and Sticky Datalog[$\exists$,=,$\perp$]
$\mathcal{ELHI}^{\neg}$ (OWL 2 EL)	Guarded Datalog[$\exists$,$\perp$]
F-Logic Lite	Weakly-Guarded Datalog[$\exists$,=] + PCC
SPARQL (with additional features)	Stratified Weakly-Guarded Datalog[$\exists$,$\neg$,$\perp$]

Implementation

- **IRIS**$^{\pm}$ – Query rewriting engine for Linear and Sticky Datalog[∃]
    - Developed at the University of Oxford
    - Several effective optimizations towards compact rewritings
    - Supports parallel rewriting
    - Database execution via SQL rewriting
    - Fully open source – https://bitbucket.org/giorsi/nyaya

Other Engines for Datalog$^{\pm}$

- ALASKA – Query rewriting engine for Linear and Sticky Datalog[∃]
  - Developed at the University of Montpellier

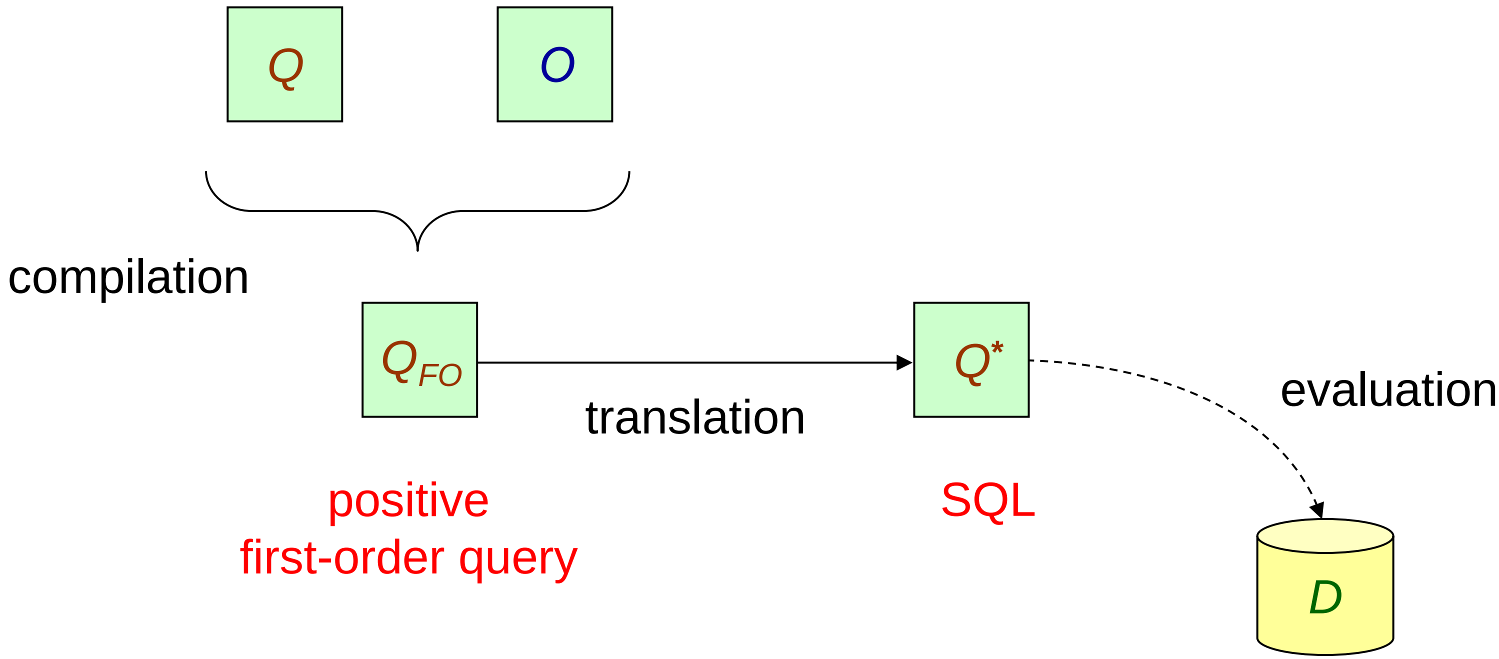
a language which captures Linear Datalog[∃] and plain Datalog

- DLV$^{\exists}$ – Query answering engine for Shy Datalog[∃]
  - Developed at the University of Calabria
  - Extension of the DLV engine

Thank you!

APPENDIX

First-Order Rewritability



$$\forall D \; (D \cup O \models Q \quad \Leftrightarrow \quad D \models Q^{*})$$

Query answering is in AC_0 in data complexity

The DL-Lite Family

Popular family of DLs with AC_0 data complexity

DL-Lite$_F$

(funct R)

DL-Lite$_R$ (OWL 2 QL)

$R \sqsubseteq E$

DL-Lite$_{core}$

$B \sqsubseteq C$

B	::=	$A \mid \exists R \mid \exists R^{-1}$	(basic concept)
C	::=	$B \mid \neg B$	(general concept)
R	::=	$P \mid P^{-1}$	(basic role)
E	::=	$R \mid \neg R$	(general role)

[Calvanese, De Giacomo, Lembo, Lenzerini & Rosati, J. Autom. Reasoning 2007]

The $\mathcal{EL}$ Family

Popular family of DLs with PTIME data complexity (OWL 2 EL)

$\mathcal{ELHI}^{\neg}$	$R \sqsubseteq E$	$E ::= R \mid \neg R$
$\mathcal{ELHI}$	$R \sqsubseteq P$	$R ::= P \mid P^{-1}$
$\mathcal{ELH}$	$P_1 \sqsubseteq P_2$	
$\mathcal{EL}$	$C_1 \sqsubseteq C_2$	$C ::= \top \mid A \mid C_1 \sqcap C_2 \mid \exists P.C$

see, e.g., [Baader, IJCAI 2003], [Rosati, DL 2007] and [Pérez-Urbina et al., J. Applied Logic 2010]

Weakly-Guarded Datalog[∃]

- All ∀-variables at affected positions occur in one body atom

null can occur
during the chase

$$\forall X \forall Y \;\; S(X,Y) \wedge P(X,Y) \rightarrow \exists Z \; P(Y,Z)$$

$$\forall X \forall Y \forall W \;\; P(X,Y) \wedge P(W,X) \rightarrow S(Y,X)$$

Affected positions = ?

[Calì, G. & Kifer, KR 2008]

Weakly-Guarded Datalog[∃]

- All ∀-variables at affected positions occur in one body atom

null can occur
during the chase

$$\forall X \forall Y \;\; S(X,Y) \wedge P(X,Y) \rightarrow \exists Z \; P(Y,Z)$$

$$\forall X \forall Y \forall W \;\; P(X,Y) \wedge P(W,X) \rightarrow S(Y,X)$$

Affected positions = {*P*[2]}

[Calì, G. & Kifer, KR 2008]

Weakly-Guarded Datalog[∃]

- All ∀-variables at affected positions occur in one body atom

null can occur
during the chase

$$\forall X \forall Y \; S(X,Y) \wedge P(X,Y) \rightarrow \exists Z \; P(Y,Z)$$

$$\forall X \forall Y \forall W \; P(X,Y) \wedge P(W,X) \rightarrow S(Y,X)$$

Affected positions = {*P*[2], *S*[1]}

[Calì, G. & Kifer, KR 2008]

Weakly-Guarded Datalog[∃]

- All ∀-variables at affected positions occur in one body atom

null can occur
during the chase

$$\forall X \forall Y \; \boldsymbol{S(X,Y)} \wedge P(X,Y) \rightarrow \exists Z\, P(Y,Z)$$

$$\forall X \forall Y \forall W \; \boldsymbol{P(X,Y)} \wedge P(W,X) \rightarrow S(Y,X)$$

Affected positions = {*P*[2], *S*[1]}

[Calì, G. & Kifer, KR 2008]

Frame Logic (F-Logic)

- Originally developed for object-oriented deductive databases
- Now used as an ontology language – Semantic Web
- F-Logic is in general undecidable

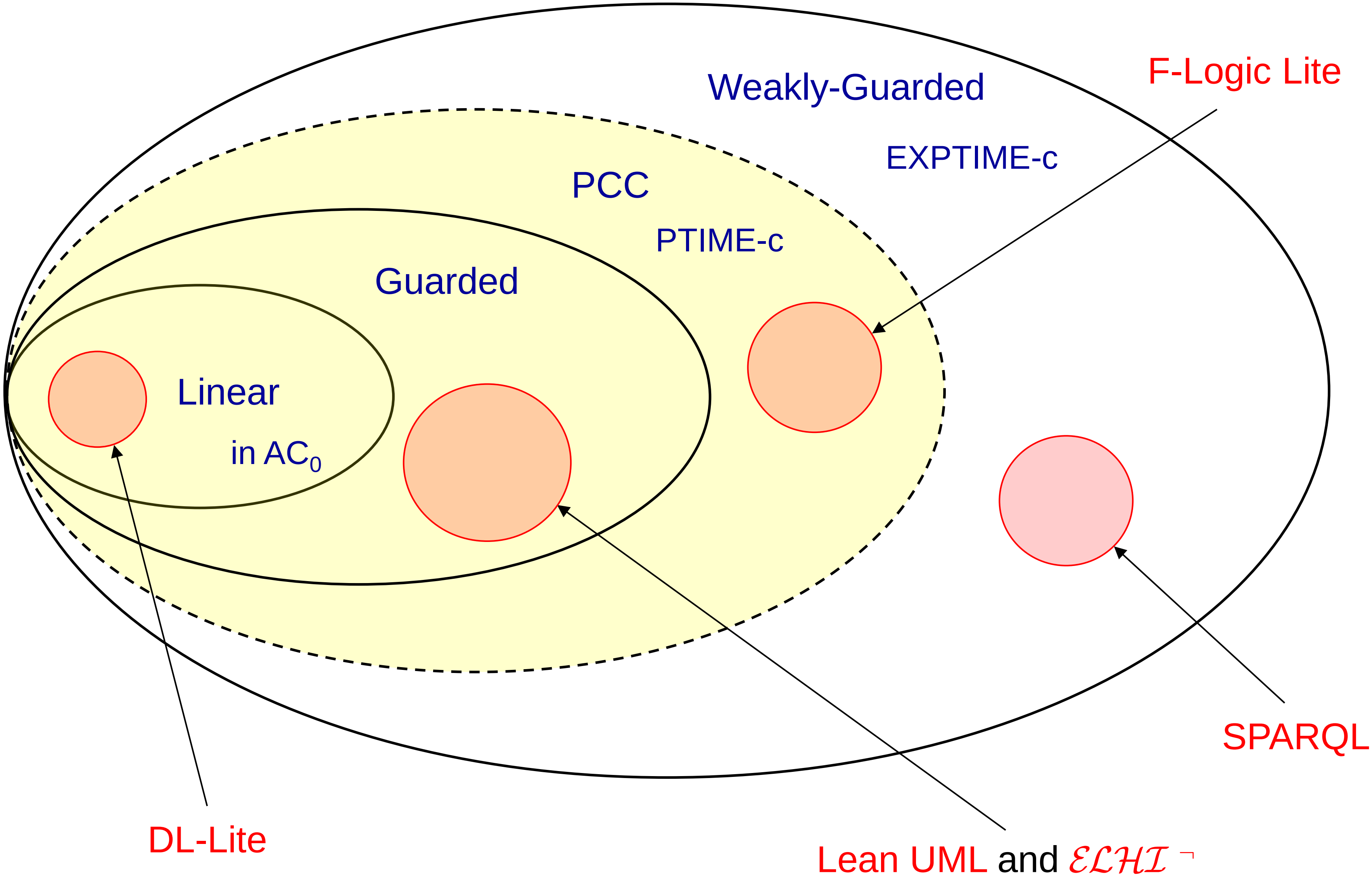
see, e.g., [Kifer, Lausen & Wu, J. ACM 1995]

F-Logic Lite

- An expressive decidable fragment of F-Logic
- Obtained from F-Logic by:
  - Excluding negation and default inheritance
  - Allowing only a limited form of cardinality constraints
- Query answering is PTIME-complete in data complexity

[Calì & Kifer, VLDB 2006]

Overview


Weakly-Guarded
EXPTIME-c
F-Logic Lite
PCC
PTIME-c
Guarded
Linear
in AC_0
SPARQL
DL-Lite
Lean UML and $\mathcal{ELHI}^{\neg}$

Complexity of Datalog[∃,=,¬,⊥]

	Linear	Guarded	Weakly-Guarded	
Arbitrary Query	PSPACE-c	2EXPTIME-c	2EXPTIME-c	Arbitrary Program
Fixed/Atomic Query	PSPACE-c	2EXPTIME-c	2EXPTIME-c	
Arbitrary Query	NP-c	NP-c	EXPTIME-c	Fixed Program
Fixed/Atomic Query	in AC_0	PTIME-c	EXPTIME-c	

Complexity of Datalog[$\exists$,=,$\neg$,$\perp$]

		Linear	Guarded	Weakly-Guarded	
Arbitrary Program	Arbitrary Query	PSPACE-c	2EXPTIME-c	2EXPTIME-c	Arbitrary Program
Arbitrary Program	Fixed/Atomic Query	PSPACE-c	2EXPTIME-c	2EXPTIME-c	Arbitrary Program
Fixed Program	Arbitrary Query	NP-c	NP-c	EXPTIME-c	Fixed Program
Fixed Program	Fixed/Atomic Query	in AC_0	PTIME-c	EXPTIME-c	Fixed Program

Data Complexity

Complexity of Datalog[∃,=,¬,⊥]

	Linear	Guarded	Weakly-Guarded	
Arbitrary Query	PSPACE-c	2EXPTIME-c	2EXPTIME-c	Arbitrary Program
Fixed/Atomic Query	PSPACE-c	2EXPTIME-c	2EXPTIME-c	
Arbitrary Query	NP-c	NP-c	NP-c	Fixed Program
Fixed/Atomic Query	in AC_0	PTIME-c	PTIME-c	

Polynomial Cloud Criterion